

Test 2 Annotated Solutions

#1. $3x^2 + xy + 4y^2 = 21$

(a) find dy/dx

(b) find eqn of line tangent to this at $(1, 2)$

(a) $6x + y + x \frac{dy}{dx} + 8y \frac{dy}{dx} = 0$

$$x \frac{dy}{dx} + 8y \frac{dy}{dx} = -6x - y \rightarrow (x + 8y) \frac{dy}{dx} = -6x - y$$

$$\rightarrow \boxed{\frac{dy}{dx} = \frac{-6x - y}{x + 8y}}$$

(b) $y - y_1 = m(x - x_1)$

$(x_1, y_1) = (1, 2), m = \left. \frac{dy}{dx} \right|_{(1,2)}$

$$y - 2 = \frac{-6(1) - 2}{1 + 8(2)} (x - 1) \rightarrow \boxed{y - 2 = -\frac{8}{17} (x - 1)}$$

#2. $C(x) = 6x^{4/3} + 60x^{2/3} + 5000$ cost per month dress prod.

Rate of production is 10 dresses/month

Find rate of cost increase at prod. level of 1000

$$\frac{dC}{dt} = \frac{dC}{dx} \cdot \frac{dx}{dt}, \quad \frac{dx}{dt} = 10 \text{ given}$$

$$= \frac{4}{3} \cdot 6x^{1/3} + \frac{2}{3} \cdot 60x^{-1/3} \cdot 10$$

$$= 8(1000)^{1/3} + 40(1000)^{-1/3} \quad x=1000 \text{ (given)}$$

$$= 80 + 40/10 = \$84/\text{mo.}$$

#3, Find derivatives.

$$f(x) = 3x^6 + 9x^3 - 12x + e + \frac{5}{x^2} - \frac{10}{x^5}$$

$$f'(x) = 18x^5 + 27x^2 - 12 + 0 - \frac{10}{x^3} + \frac{50}{x^6}$$

$$y = \frac{\ln x}{5x^3 + x + 1} \quad y' = \frac{\frac{1}{x}(5x^3 + x + 1) - \ln x(15x^2 + 1)}{(5x^3 + x + 1)^2}$$

$$y' = \frac{5x^2 + 1 + \frac{1}{x} - 15x^2 \ln x - \ln x}{(5x^3 + x + 1)^2}$$

$$h(x) = \frac{5}{(2x^3 + 1)^8} = 5(2x^3 + 1)^{-8} \quad \leftarrow \text{easier to convert to chain rule w/ quotient}$$

$$h'(x) = -40(2x^3 + 1)(6x^2) = -240x^2(2x^3 + 1)$$

$$s(x) = e^{\sqrt{3x^2 + 2x - 8}}, \quad s'(x) = e^u \frac{du}{dx}$$

$$s'(x) = e^{(\sqrt{3x^2 + 2x - 8})^{1/2}} \cdot \frac{1}{2}(3x^2 + 2x - 8)^{-1/2} (6x + 2)$$

$$= \frac{e^{\sqrt{3x^2 + 2x - 8}} (6x + 2)}{2\sqrt{3x^2 + 2x - 8}}$$

$$p(t) = \sqrt[5]{200 + 4^t} = (200 + 4^t)^{1/5}$$

$$\text{By chain rule: } p'(t) = \frac{1}{5}(200 + 4^t)^{-4/5} (4^t \ln 4)$$

$$y(x) = \left(\frac{2x-1}{9x+4}\right)^6 \quad \text{power + chain + quotient rule}$$

$$y'(x) = 6 \left(\frac{2x-1}{9x+4}\right)^5 \left(\frac{2(9x+4) - (2x-1)9}{(9x+4)^2}\right) = 6(17) \frac{(2x-1)^5}{(9x+4)^7}$$

$$y = e^{10x^2}, \quad y' = 20xe^{10x^2}$$

$$f(x) = 12x^9 - 8x^7 - x^6 + 2x^5 - 17x^4 + 32x^3 - 11x + 105$$

$$f^{(20)}(x) = 0 \quad \text{since } f'(x) = \sim x^8, \quad f''(x) = \sim x^7, \dots$$

$$\dots f^{(8)} = \sim x, \quad f^{(9)} = \text{const}, \quad f^{(10)+} = 0$$

#4. Find the critical numbers. Be sure you can define what is meant by a critical number.

a) $f(x) = 2x^3 - 6x + 13$ Dom: \mathbb{R}

b) $g(x) = \frac{x^2 + 7}{x + 3}$ Dom: $x \neq -3$

c) $h(x) = x^{2/3}(4-x)$ Dom: \mathbb{R}

a) $f'(x) = 6x^2 - 6 = 6x(x-1) = 0$ at $\boxed{x=0, 1}$

b) $g'(x) = \frac{2x(x+3) - (x^2+7)(1)}{(x+3)^2}$

Must simplify to find crit #

$$= \frac{2x^2 + 6x - x^2 - 7}{(x+3)^2} \leftarrow \text{Set numerator} = 0$$

$$\rightarrow x^2 + 6x - 7 = (x-1)(x+7) = 0, \quad \boxed{x=1, -7}$$

Is $x = -3$ a crit #? No! It is not in the domain.

c) $h'(x) = \frac{2}{3}x^{-1/3}(4-x) + x^{2/3}(-1)$

$$= \frac{2(4-x)}{3x^{1/3}} - x^{2/3} = \frac{2(4-x) - 3x^{2/3}x^{1/3}}{3x^{1/3}} \leftarrow \text{LCD}$$

$$h(x) = \frac{8-5x}{3x^{2/3}} \quad \text{Crit \#s: } 8-5x=0 \rightarrow \frac{8}{3} = 0$$

$$\boxed{x = 8/5} \quad \boxed{x = 0}$$

Both are in domain of $h(x)$ so they're both crit #'s.

#5) $q(p) = -150p + 750$

a) Find $R(p)$, i.e. $R = pq$ in terms of p :

$$R(p) = p \cdot q(p) = p(-150p + 750)$$

$$\boxed{R(p) = 750p - 150p^2}$$

b) Find dR/dp , i.e. $R'(p)$, marginal revenue

$$\boxed{R'(p) = 750 - 300p}$$

c) Find dR/dq , i.e. $R'(q)$

First we need R in terms of q . Solve

$$q = -150p + 750 \quad \text{for } p: \quad p = \frac{750 - q}{150}$$

$$R(q) = q \left(\frac{750 - q}{150} \right) \quad \boxed{R'(q) = 5 - \frac{2q}{150}}$$

d) $R'(q)$ at $q = 225 = 5 - \frac{2(225)}{150} = 2$

For the $q+1 = 226$ th item sold,
the additional revenue is \$2.

-that is

#6) Position of particle: $x(t) = t + \frac{4}{t+1}$, $t \geq 0$

Find position at $t = 5$ sec

$$x(5) = 5 + \frac{4}{6} = \boxed{5\frac{2}{3} \text{ m}}$$

Velocity at $t = 2$ sec

$$x'(t) = 1 - 4(t+1)^{-2} = 1 - \frac{4}{(t+1)^2}$$

$$x'(2) = 1 - \frac{4}{9} = \boxed{5/9 \text{ m/sec}}$$

Direction of particle at $t = 0.5$ sec

$$x'(.5) = 1 - \frac{4}{(1.5)^2} = 1 - \frac{4}{2.25} < 0$$

Since x' is negative, particle is going in reverse direction (relative to going forward being positive velocity).

$$a(t) = x''(t) = \frac{+8}{(t+1)^3}, \quad x''(1) = \frac{8}{2^3} = \frac{8}{6}$$

$$\boxed{a(1) = \frac{4}{3} \text{ m/sec}^2}$$

#7) a)



On $[a, b]$, a cts fun. has a max + a min (EVT)

b) $f'(c) = \frac{f(b) - f(a)}{b - a}$ for some $c \in [a, b]$ if f is differentiable on (a, b)

c) $f'(c) = 0$, $f'(c) \text{ DNE} \Leftrightarrow c$ is crit. \neq

d) $f(a) \leq f(x)$ for all x near $a \Rightarrow f$ has local min at $x = a$

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