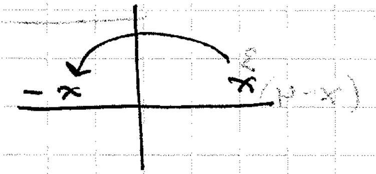
Test 2 Solutions $\overline{w}$ notes Fall 2014 McKenzie

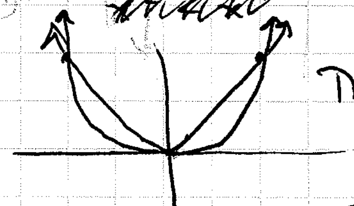
1. Negating x-values transforms graph over y-axis.



Range of $f(x) = \frac{1}{x}$ is $x \neq 0$, i.e. $(-\infty, 0) \cup (0, \infty)$

Domain $f(x) = \sqrt[4]{x}$ is $x \geq 0$, i.e. $[0, \infty)$

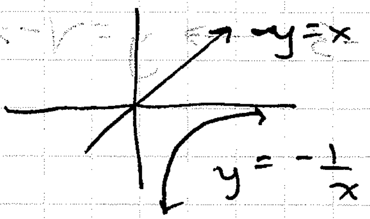
For $0 < x < 1$, which is not true?

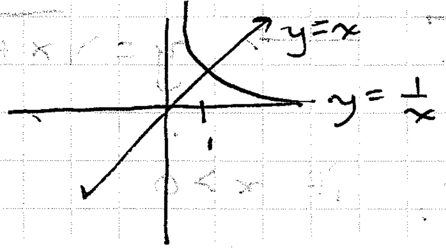


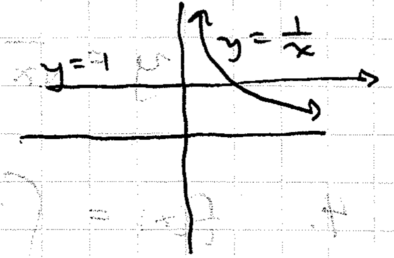
The wider one is x^4, until it reaches $x = 1$. Then it climbs faster than x^2.

So $x^2 < x^4$ is not true

The others are true on this interval:







Compress f vertically by a factor of 2: $\frac{1}{2} f(x)$

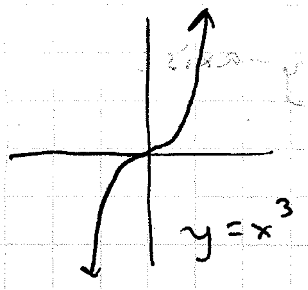
Climbs slower $\nearrow$

$f(x) = \sqrt[5]{x}$ is symmetric with respect to origin

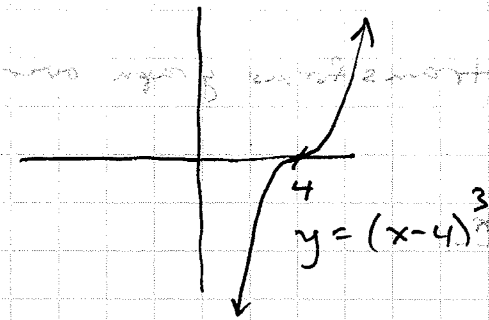
(An odd fcn: $f(-x) = -f(x)$ since

$$\sqrt[5]{-x} = \sqrt[5]{-1}\sqrt[5]{x} = -1\sqrt[5]{x}$$

2. $g(x) = -2(x-4)^3 + 1$

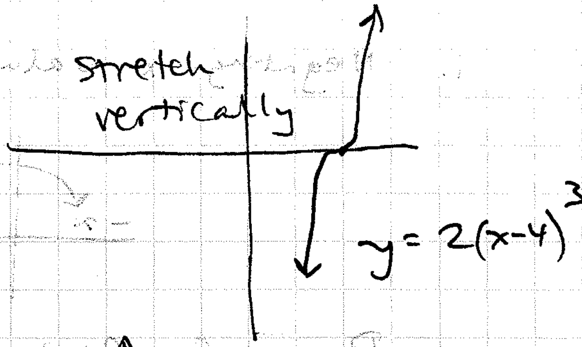


$y = x^3$

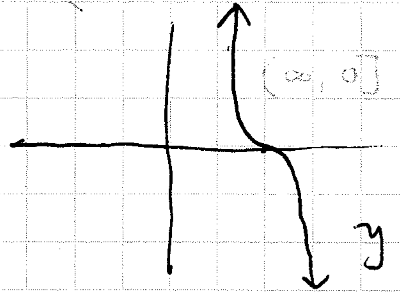


$y = (x-4)^3$

stretch vertically

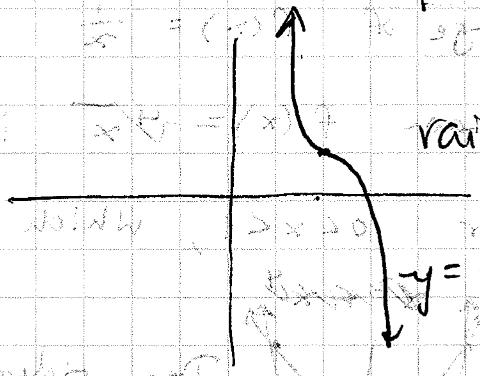


$y = 2(x-4)^3$



flip over x-axis

$y = -2(x-4)^3$



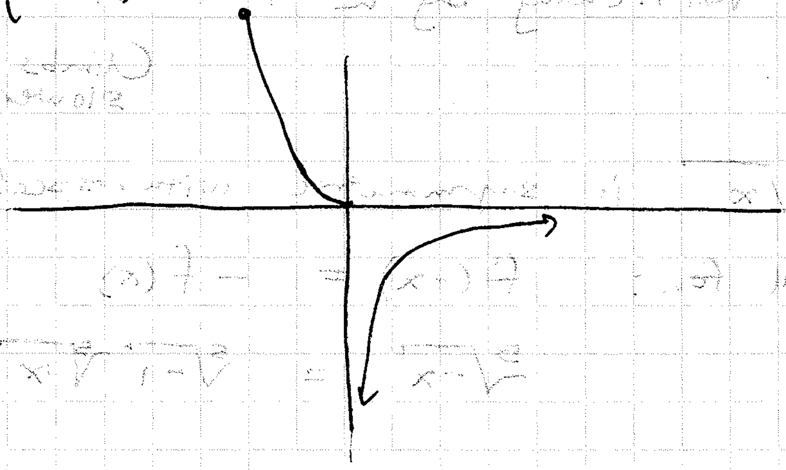
raise up 1

$y = -2(x-4)^3 + 1$

3. Shift $y = \sqrt{x}$ 2 units left, 3 units down, before reflecting over y-axis.

$$y = \sqrt{x+2} \longrightarrow y = \sqrt{x+2} - 3 \longrightarrow y = \sqrt{-x+2} - 3$$

4. $$f(x) = \begin{cases} -\frac{1}{x}, & \text{if } x > 0 \\ x^2, & \text{if } -2 < x \leq 0 \end{cases}$$



5. $A = (2,5) \qquad B = (-1,7)$

a) $m = \dfrac{5-7}{\cancel{2+1}\;2-{}^-1} = \dfrac{-2}{+3} = -\dfrac{2}{3}$

$y - y_1 = m(x - x_1)$

$y - 5 = -\frac{2}{3}(x-2)$

$y = -\frac{2}{3}x + \frac{19}{3}$

b) $y = -\frac{3}{2}x + 5$

$m_\perp = -\left(\frac{1}{-3/2}\right) = \frac{2}{3}$

$y - 5 = \frac{2}{3}(x-2)$

$y = \frac{2}{3}x + \frac{11}{3}$

c) Vert line thru $B = (-1,7)$: $x = -1$

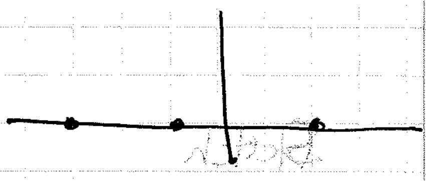
d) Midpt $= \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{2+{}^-1}{2}, \frac{5+7}{2}\right) = \left(\frac{1}{2}, 6\right)$

e) dist from A to B $= \sqrt{(x_1-x_2)^2 + (y_1-y_2)^2}$

$= \sqrt{(2-{}^-1)^2 + (5-7)^2} = \sqrt{13}$

6. Polynomial fcn $f(x)$ has properties below:

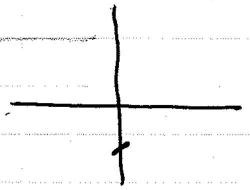
$x = -3, -1, 2$ only zeroes (roots)



As $x \to \infty$, $f(x) \to -\infty$ ↘

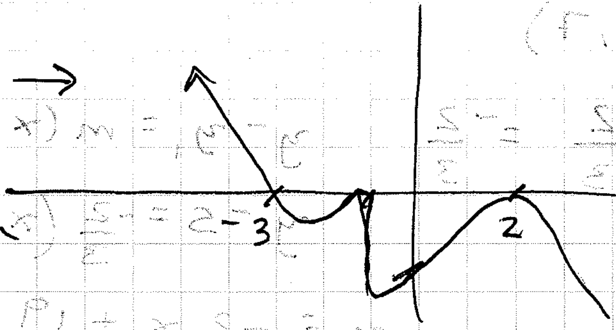
As $x \to -\infty$, $f(x) \to \infty$ ↖

$f(0) < 0$ (i.e., y-int is negative)

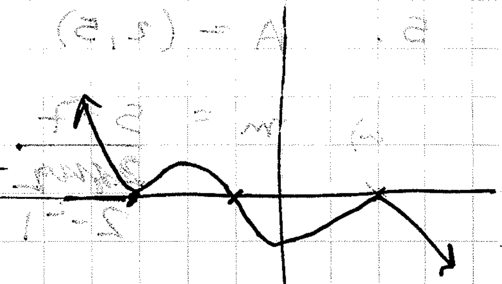


Combine these to find a possible graph

Need these → to come up with a function



or



$$f(x) = -(x+3)(x+1)^2(x-2)^2$$

or $$f(x) = -(x+3)^2(x+1)(x-2)^2$$

7. $g(x) = \overset{2}{-}\frac{1}{3}(x-1)^2(2\overset{1}{x}-5)(\overset{1}{x}+2)$

deg: 4 l.c.: $(-\frac{1}{3})(2) = -\frac{2}{3} < 0$

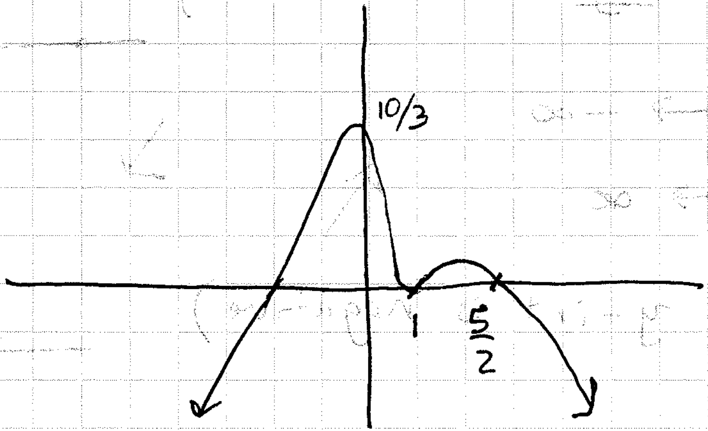
End behavior ↙ ↘ even deg, neg l.c.

y-int: $g(0) = -\frac{1}{3}(-1)^2(-5)(2) = \frac{10}{3}$ $(0, \frac{10}{3})$

x-int: $g(x) = 0 = -\frac{1}{3}(x-1)^2(2x-5)(x+2)$

$x = 1, \frac{5}{2}, -2$, so $(1,0), (\frac{5}{2},0), (-2,0)$ are the roots

Sketch



x=1 is location of the double rot

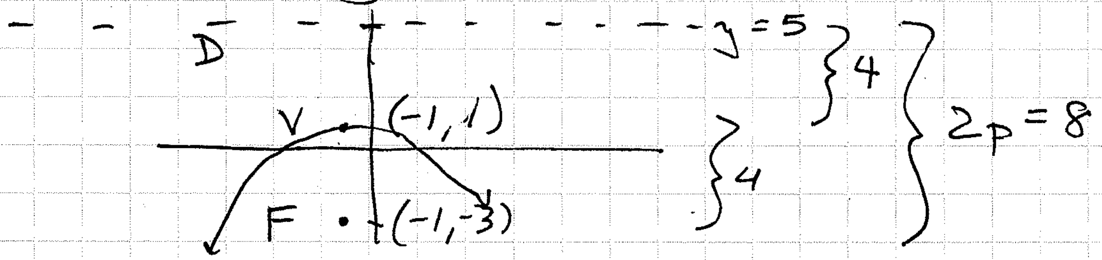
8.a) Parabola focus $(-1,-3)$ Directrix $y=5$

Find the equation: $y-k = \frac{1}{4p}(x-h)^2$

The dist between focus y value + directrix $= 2p$

so $2p = |-3-5| = 8$, hence $p=4$

A sketch finds us the vertex, since it is on the halfway pt. between -3 and 5, or $y=1$



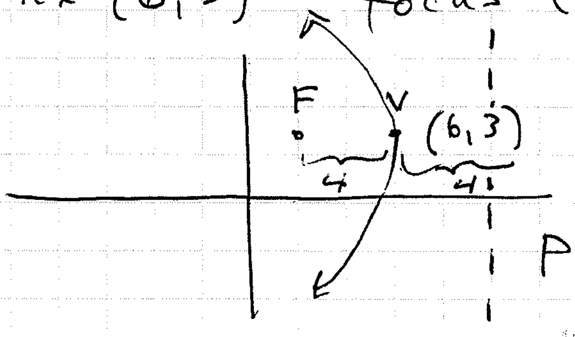
Before sketching, I complete the eqn. (It doesn't ~~ask~~ ask for the sketch, but visualizing helped me find vertex.

$$y-1 = \frac{1}{4\cdot 4}(x-{}^-1)^2$$

$$\boxed{y-1 = \frac{1}{16}(x+1)^2}$$

It's compressed vertically b/c $p>1$

b) Vertex $(6,3)$ focus $(2,3)$



The sketch shows it is sideways, left opening.

$p = 6-2 = 4$

$\frac{1}{4p} = \frac{1}{16}$

Directrix is to the right 4 units from vertex:

$$\boxed{x=10}$$

$$x-h = \frac{-1}{4p}(y-k)^2$$

$$x-6 = \frac{-1}{16}(y-3)^2$$ (not asked for)

c) $h(x) = \frac{3x^2 + 2x + 8}{8x^2 + 7}$ $\frac{\deg 2}{\deg 2} \Rightarrow \boxed{HA: y = \frac{3}{8}}$

↖ lead coeff ratio

$\boxed{\text{VA: Since } 8x^2 + 7 \neq 0 \text{, there is none}}$, $\boxed{\text{No slant}}$

#11) $f(x) = \frac{3(x+2)^2 (x-4)(x+3)}{(x+3)(x-2)^3}$

$\boxed{\text{Dom: } x \neq -3, 2}$

y-int: $f(0) = \frac{\cancel{3}(4)(-4)(3)}{\cancel{3}(-8)} = 6$ y-int $\boxed{(0, 6)}$

x-int: (roots) $3(x+2)^2 (x-4) \cancel{(x+3)} = 0$

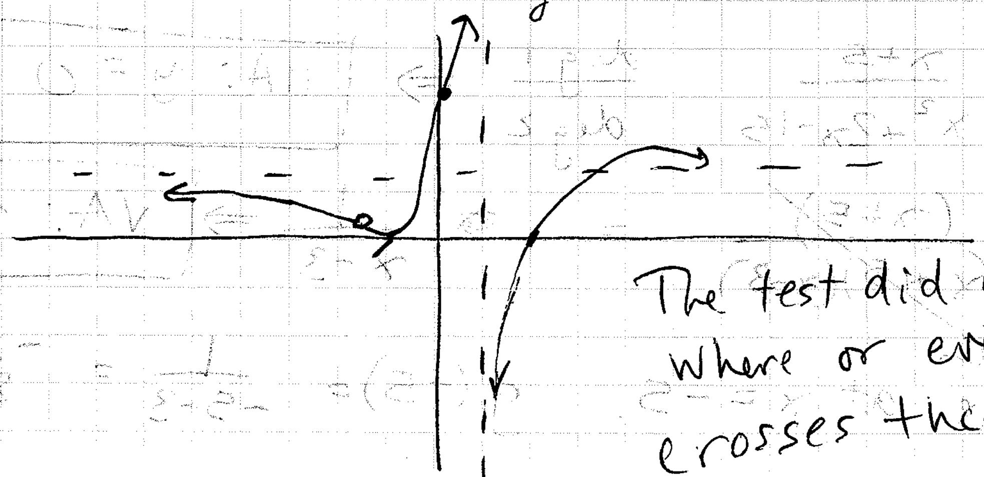
$x = -2, 4, \cancel{-3}$ $\boxed{(-2,0), (4,0), \cancel{(-3,0)}}$

↑ This is a hole coordinate

hole: $x = -3$

$f(-3) = \frac{3(-3+2)^2(-3-4)}{(-3-2)^3} = \frac{21}{125}$ $\boxed{\left(-3, \frac{21}{125}\right) \text{ hole}}$

$\boxed{\text{VA: } x = 2}$ HA: $\frac{\deg 4}{\deg 4} \Rightarrow y = \frac{3}{1}$ $\boxed{y_{HA} = 3}$



The test did not ask where or even if it crosses the HA.

~~b)~~ #9) a) $2x^4 - 5x^2 - 12 = 0$

$$(2x^2+3)(x^2-4)=0$$

$$\underset{0 \text{ in } \mathbb{R}}{\neq} \qquad \underset{0}{=} \text{ when } \boxed{x = \pm 2}$$

i.e. $2x^2+3=0$

$$x^2 = -\frac{3}{2} \longrightarrow \boxed{x = \pm\sqrt{-\frac{3}{2}}}$$

b) $2x^2 + x - 5 = 0$ Doesn't factor easily (no rational roots)

QF: $x = \dfrac{-1 \pm \sqrt{1^2 - 4(2)(-5)}}{2(2)} = \boxed{\dfrac{-1 \pm \sqrt{41}}{4}}$

c) $(3x+2)^2 = 7$

$$3x+2 = \pm\sqrt{7} \longrightarrow \boxed{x = \frac{-2 \pm \sqrt{7}}{3}}$$

#10) a) $f(x) = \dfrac{2x^2 - x - 7}{x-2}$ $\qquad \dfrac{\deg 2}{\deg 1} \Rightarrow$ SA

$$\begin{array}{r} 2x + 3 \\ x-2 \overline{\big)\, 2x^2 - x - 7} \\ -(2x^2 - 4x) \\ \hline 3x - 7 \end{array} \longrightarrow \boxed{y = 2x+3 \text{ is SA}}$$

$x - 2 \neq 0$ so $\boxed{x = 2 \text{ is VA}}$

b) $g(x) = \dfrac{x+5}{x^2+2x-15}$ $\qquad \dfrac{\deg 1}{\deg 2} \Rightarrow \boxed{\text{HA: } y = 0}$

$$= \frac{\cancel{(x+5)}}{\cancel{(x+5)}(x-3)} = \frac{1}{x-3} \Rightarrow \boxed{\text{VA: } x = 3}$$

(Not asked for) ← Hole at $x = -5$. $\quad g(-5) = \dfrac{1}{-5-3} = -\dfrac{1}{8}$ $\quad \left(-5, -\dfrac{1}{8}\right)$