Test 1 Math 220 - Sec 4 Solutions

(+1) 1.a) $x^2+1 \geq 0 \rightarrow \boxed{x \in \mathbb{R}}$

(+5) b) $f(0) = 3 - \sqrt{0+1} = 3 - 1 = 2$ $\boxed{(0,2) \text{ y-int}}$

$f(x) = 0 = 3 - \sqrt{x^2+1} \longrightarrow 3 = \sqrt{x^2+1} \longrightarrow 9 - 1 = x^2$, $8 = x^2$ $\boxed{x = \pm\sqrt{8}}$ x-ints.

2. Find the limits using algebra or, for (a), a graph + interpretation.

(5) 2. a) $\lim_{x \to 2} \frac{x+2}{x^2-4} = \lim_{x \to 2} \frac{1}{x-2} = \frac{1}{0} = \infty$

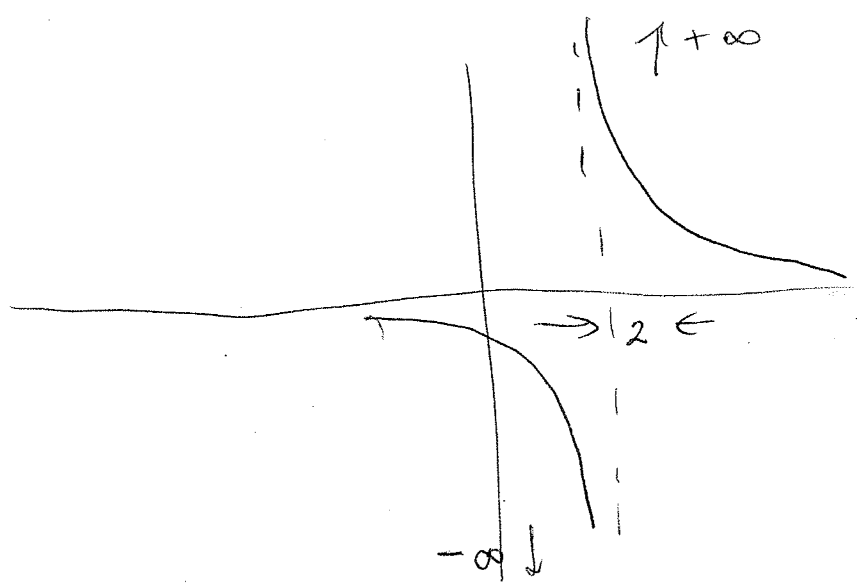
but, there are 2 sides to consider

$\lim_{x \to 2^+} \left(\frac{1}{x-2}\right) = \frac{1}{\text{small, positive}} = +\infty$

and

$\lim_{x \to 2^-} \left(\frac{1}{x-2}\right) = \frac{1}{\text{small negative}} = -\infty$

$\left.\begin{matrix} \\ \end{matrix}\right\}$ LHL ≠ RHL so $\lim_{x \to 2} \frac{x+2}{x^2-4}$ DNE



+5 b) $\lim_{h\to 0} \frac{\sqrt{h+9}-3}{h} = \frac{0}{0}$ indeterminate; need algebra

$$\lim_{h\to 0} \frac{\sqrt{h+9}-3}{h} \cdot \frac{\sqrt{h+9}+3}{\sqrt{h+9}+3} = \lim_{h\to 0} \frac{h+9-9}{h(\sqrt{h+9}+3)}$$

$$= \lim_{h\to 0} \frac{\cancel{h}}{\cancel{h}(\sqrt{h+9}+3)} = \lim_{h\to 0} \frac{1}{\sqrt{h+9}+3} = \frac{1}{\sqrt{9}+3} = \boxed{\frac{1}{6}}$$

3. a) +2 $C(50) = 3(50) + b = 270 \implies \boxed{b = 120}$

+1 $\boxed{C(x) = 3x + 120}$

b) b is the fixed production cost, that is, the
+2 expense each day for before producing any tortillas (e.g. rent, utilities)

+2 c) $\boxed{R(x) = 5x}$

d) +1 $P(x) = R(x) - C(x)$

+1 $\boxed{P(x) = 2x - 120}$

+2 e) $P(x) = 0 = 2x - 120$ when $\boxed{x = 60 \text{ packages}}$

+2 f) $P'(x) = \$2$, the profit ~~from~~ from each additional package sold.
+2

4. $m = f'(x) = \dfrac{-1}{(x-3)^2}$ (+1) (+1) $\quad m = f'(1) = -\dfrac{1}{4}$ (+1)

(+1) $y - y_1 = m(x - x_1)$ $\quad y_1 = f(x_1) = \dfrac{1}{1-3} = -\dfrac{1}{2}$ (+1)

(+2) $\boxed{y + \frac{1}{2} = -\frac{1}{4}(x-1)}$

5. (+2) a) $f'(x) = 3(x^4 - 5x^2 + 7)^2(4x^3 - 10x)$

(+4) total b) $f'(x) = \dfrac{2(x-3)\cdot 6e^{3x} - (x-3)^2\, 6e^{3x}\cdot 3}{(6e^{3x})^2}$ (+1) (+1) (+1)

(+1) $= \dfrac{12(x-3)e^{3x} - 18e^{3x}(x-2)^2}{(6e^{3x})^2}$ or further simplification

6a) (+2) $P'(x) = 120 \cdot \frac{2}{3}x^{-1/3} + 9 \cdot \frac{1}{3}x^{-\cancel{2}4/3} = \boxed{\dfrac{80}{x^{1/3}} + \dfrac{3}{x^{4/3}}}$

(+1) $P'(8) = \dfrac{80}{8^{1/3}} + \dfrac{3}{8^{\cancel{2}4/3}}$

(+2) $= \dfrac{80}{2} + \dfrac{3}{2^2} = 40 + \dfrac{3}{4} = \$40.75$ (+1)

b) $P'(8)$ is the increase in profit from the 8th to the 9th lamp. (Not "the profit from the 9th") (+2)

Note $P(9) - P(8) \approx P'(8)$