Math 108 Fall '14

Test 1 Solutions with notes

\# 1. Simplify - eliminate negative exponents

a) $(3x^6y^{11})(4x^{-19}y^4) = (3)(4)(x^6x^{-19})(y^{11}y^4) = 12x^{-13}y^{15}$

add exps of like bases

$$= \boxed{\frac{12y^{15}}{x^{13}}}$$

b) $\dfrac{(2x)^0}{-2^3} = \dfrac{1}{-(2\cdot 2\cdot 2)} = \dfrac{1}{-8} = \boxed{-\dfrac{1}{8}}$

Notice the negative is not raised to the power since it is not enclosed in ( ) at the start

c) $\dfrac{(-8x^2y^2)^4}{(16x^3y^7)^2}$ use rule $(a^m)^n = a^{mn}$ $\rightarrow \dfrac{(-8)^4x^8y^8}{16^2x^6y^{14}}$

$$= \frac{(-1)^4(2^3)^4x^8y^8}{(2^4)^2x^6y^{14}} = \frac{2^{12}x^8y^8}{2^8x^6y^{14}}$$

Notice that 8 + 16 are powers of 2, which simplifies the process (even power of -1 is positive).

$$= 2^{12-8}x^{8-6}y^{8-14}$$

$$= 2^4x^2y^{-6} = \boxed{\frac{16x^2}{y^6}}$$

2. a) $\sqrt[3]{250p^9q^4} = \sqrt[3]{250}\,\sqrt[3]{p^9}\,\sqrt[3]{q^4}$ I broke it up to make it clear

first part $= \sqrt[3]{125\cdot 2} = \boxed{5\sqrt[3]{2}}$ second part $(p^9)^{1/3} = \boxed{p^3}$

know your powers of 2, 3, 5, ...

third part $\sqrt[3]{q^4} = \sqrt[3]{q^3}\,\sqrt[3]{q} = \boxed{q\sqrt[3]{q}}$ Glue it back together now

$5\sqrt[3]{2}\; q\sqrt[3]{q}\; p^3 = \boxed{5p^3q\sqrt[3]{2q}}$ final

b) $\sqrt{300} + \sqrt{27} = \sqrt{3\cdot 100} + \sqrt{3\cdot 9} = 10\sqrt{3} + 3\sqrt{3} = \boxed{13\sqrt{3}}$

c) $(2\sqrt{5}-3)(3\sqrt{5}+4) = 6\sqrt{5}^2 + 8\sqrt{5} - 9\sqrt{5} - 12$

FOIL this

$= 6\cdot 5 - \sqrt{5} - 12$

$= 30 - 12 - \sqrt{5} = \boxed{18\sqrt{5}}$

3 – Factor the expressions (next page)

The types of factoring are

- common factoring
- Special products ⇒ (diff of cubes, $a^3 - b^3$
  sum of cubes, $a^3 + b^3$
  diff of squares, $a^2 - b^2$
  but no sum of squares)
  $a^2 + b^2 \neq (a+b)^2$
- factor by grouping
- FOIL backwards

#3. a) $125x^3 - 27y^3 = (5x)^3 - (3y)^3$

It's a difference of cubes.

Know the formula: $a^3 - b^3 = (a-b)(a^2+ab+b^2)$

where $a = 5x$ and $b = 3y$ here.

$$(5x)^3 - (3y)^3 = (5x-3y)((5x)^2 + (5x)(3y) + (3y)^2)$$

$$= \boxed{(5x-3y)(25x^2 + 15xy + 9y^2)}$$

b) $12x^3 - 26x^2 - 30x$ common factor $2x$

$= 2x(6x^2 - 13x - 15)$ reverse FOIL (by trial + error)

$= \boxed{2x(6x+5)(x-3)}$ Always check your ans.

c) $6xy - 21y + 2xz - 7z$ - 4 terms make me think of grouping terms first

$= ((6xy - 21y) + (2xz - 7z))$. common factor from each ( )

$= \cancel{3y\ 3x}\ 3y(2x-7) + z(2x-7)$. again, common factor of both is $2x-7$

$= \boxed{(2x-7)(3y+z)}$

#4) Perform the operations on the rational expressions. This requires factoring so you can see what reduces (cancels); or factor to discover the LCD.

a) $$\frac{2x-3}{x+5} - \frac{x^2-4x-19}{x^2+8x+15} = \underbrace{\frac{2x-3}{(x+5)} - \frac{x^2-4x-19}{(x+5)(x+3)}}$$

LCD is on right

$$= \frac{(x+3)}{(x+3)} \cdot \frac{(2x-3)}{(x+5)} - \frac{(x^2-4x-19)}{(x+5)(x+3)}$$

$$= \frac{2x^2-3x+6x-9-x^2+4x+19}{(x+3)(x+5)}$$

FOIL the left
Dist. the negative on the right

$$= \frac{2x^2-x^2-3x+4x+6x-9+19}{(x+3)(x+5)}$$

Combine like terms

$$= \frac{x^2+7x+10}{(x+3)(x+5)}$$

Factor the numerator

$$= \frac{(x+2)(x+5)}{(x+3)(x+5)}$$

At this point, notice that $x \neq -3$ or -5, regardless of the final answer's reduced form.

$$= \boxed{\frac{x+2}{x+3} \quad \text{where} \quad x \neq -5}$$

← State the only the restriction that is not apparent in final ans.

#4b) $\dfrac{x^2}{4x^2+12x} \div \dfrac{x-3}{x^2-9}$ Invert + multiply

$= \dfrac{x^2}{4x^2+12x} \cdot \dfrac{x^2-9}{x-3}$ Factor expressions as needed

$= \dfrac{x^2}{4x(x+3)} \cdot \dfrac{(x+3)(x-3)}{(x-3)}$ Reduce common factors

$= \dfrac{x^{\cancel{2}}\cancel{(x+3)}\cancel{(x-3)}}{4\cancel{x}\cancel{(x+3)}\cancel{(x-3)}} = \boxed{\dfrac{x}{4}, \quad x \neq 0, 3, -3}$

#5. Solve for x – There is a variety of problem types. The goal is to isolate x. Check answer for so-called "extraneous solutions"

a) $\dfrac{100-4x}{3} = \dfrac{5x+6}{4} + 6$ ⊗ both sides by LCD (i.e., "cross-multiply")

$\cancel{12}^{4}\dfrac{(100-4x)}{\cancel{3}} = \dfrac{(5x+6)}{\cancel{4}}\cancel{12}^{3} + 6^{(12)}$ Reduce

$400 - 16x = 15x + 18 + 72$ Expand

Combine like terms + solve

$310 = 31x$

$\boxed{x = 10}$

\# 5b) $\sqrt{x+3} - \sqrt{2x+4} = -1$

Best to place $\sqrt{\ }$ terms on opposite sides so you won't have a lot of $\sqrt{\ }$'s to FOIL

$$\sqrt{x+3} = \sqrt{2x+4} - 1$$

$$\left(\sqrt{x+3}\right)^2 = \left(\sqrt{2x+4} - 1\right)^2$$

Square both sides

$$x+3 = 2x+4 - \sqrt{2x+4} - \sqrt{2x+4} + 1$$

FOIL - do not make the so-called freshman error $(a-b)^2 \neq a^2 - b^2$

$$x+3 = 2x+4+1 - 2\sqrt{2x+4}$$

$$x - 2x + 3 - 4 - 1 = -2\sqrt{2x+4}$$

$$-x-2 = -2\sqrt{2x+4}$$

Cancel -1 across

$$x+2 = 2\sqrt{2x+4}$$

Now square both sides again

$$(x+2)^2 = 4(2x+4)$$

Expand left by FOIL

$$x^2+2x+2x+4 = 8x+16$$

$$x^2 - 4x - 12 = 0$$

Set $= 0$. Factor + solve

$$(x-6)(x+2) = 0$$

$$x = 6, -2$$

Check both into original equation.

$x=6$: $\sqrt{6+3} - \sqrt{12+4} \stackrel{?}{=} -1$

$3 - 4 = -1$

checks

$x=-2$: $\sqrt{-2+3} - \sqrt{-4+4} \stackrel{?}{=} -1$

$1 - 0 \neq -1$

discard $x=-2$

$$\boxed{x = 6}$$

5c) $x(x+2)=99$ Set it all = zero

$x(x+2)-99=0$ Expand

$x^2+2x-99=0$ Factor

$(x-9)(x+11)=0$

$\boxed{x=9,\ -11}$ Both check out.

d) $x^{-1}-9x^{-3}=0$

2 approaches:

A Factor out one of the x terms

or B Write as fractions + get LCD

A $\underbrace{x^{-3}}_{=\text{zero}}\ \text{or}\ \underbrace{(x^2-9)}_{=\text{zero}}=0$ (since $x^{-1}/x^{-3}=x^2$)

$\frac{1}{x^3}\neq 0$ since $1\neq 0$ or $x^2-9=0$

$\boxed{x=3,-3}$ is good (both check)

B $\frac{1}{x}-\frac{9}{x^3}=0$ LCD: x^3

$\frac{x^2}{x^3}-\frac{9}{x^3}=\frac{x^2-9}{x^3}=0$ when $x^2-9=0$

or $\boxed{x=\pm 3}$

#6 Polynomial division of

$8x^3+10x+1$ by $2x+3$,

by which we discover if $2x+3$

is a factor of $8x^3+10x+1$.

$$
\begin{array}{r}
4x^2 - 6x + 14 \\
2x+3 \,\overline{\big)\, 8x^3 + 0x^2 + 10x + 1} \\
-(8x^3 + 12x^2) \\
\hline
-12x^2 + 10x \\
-(-12x^2 - 18x) \\
\hline
28x + 1 \\
-(28x + 42) \\
\hline
-41
\end{array}
$$

← remainder term

Ans: $\boxed{4x^2 - 6x + 14 + \frac{-41}{2x+3}}$

So $2x+3$ is not a factor of $8x^3+10x+1$.

#7. Complete the square to rewrite

$-5x^2 + 30x - 41$ in form of

$a(x-h)^2 + k$ ← where $h = \frac{b}{2}$ of factored $x^2 + bx$

$$-5x^2 + 30x - 41 = \underbrace{-5(x^2 + 6x)}_{\text{factor out from first 2 terms only}} - 41$$

$$= -5\left(x^2 + 6x + \left(\frac{6}{2}\right)^2\right) - 41 + ??$$

(note: $x^2 + 6x$ is where we get $b = 6$)

You have to undo whatever you did by adding $\left(\frac{b}{2}\right)^2$, including the $\otimes$ by -5

$\left(\frac{b}{2}\right)^2 = 3^2 = 9$, but $(-5)(9) = -45$

$$= -5(x+3)^2 - 41 + 45$$

$$= \boxed{-5(x+3)^2 + 4}$$

#8. Find domain, express in interval notation.

a) $f(x) = \begin{cases} -3x+2, & \text{if } x \geq 5 \\ x^2 - 5x - 11, & \text{if } x < -1 \end{cases}$ ← Here's the domain in typical algebraic form

Here's the graph

$\boxed{(-\infty, -1) \cup [5, \infty)}$ final

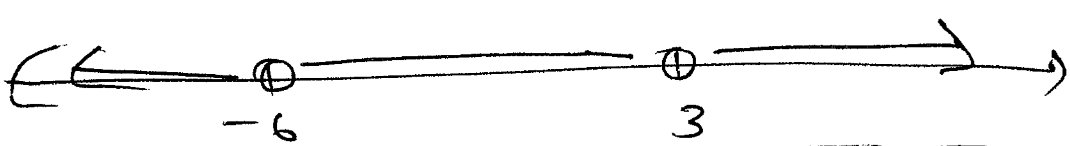
b) $g(x) = \sqrt[6]{x+3}$ The radicand must be nonnegative (i.e., zero or pos.) since the root is even

$x+3 \geq 0 \Rightarrow x \geq -3$ $\boxed{[-3, \infty)}$

c) $h(x) = \dfrac{x^2 - 2x - 3}{x^2 + 3x - 18}$ $\Rightarrow$ Denom $\neq 0$ so set $\neq 0$ + solve

$x^2 + 3x - 18 \neq 0$

$(x-3)(x+6) \neq 0$ $x \neq 3, -6$



$\boxed{(-\infty, -6) \cup (-6, 3) \cup (3, \infty)}$ ← Ans.

#9. a) Use the def. of a one-one function to show $f(x) = 2\sqrt[3]{x-5} + 1$ is 1-1.

This entails making the assumption that for $a, b \in \text{dom } f(x)$, $f(a) = f(b)$ and from there showing algebraically that $a = b$.

$$f(a) = f(b)$$

$$2\sqrt[3]{a-5} + \cancel{1} = 2\sqrt[3]{b-5} + \cancel{1}$$
$$\underline{\quad -1 \quad} \qquad \underline{\quad -1 \quad}$$

$$\frac{\cancel{2}\sqrt[3]{a-5}}{\cancel{2}} = \frac{\cancel{2}\sqrt[3]{b-5}}{\cancel{2}}$$

$$\sqrt[3]{a-5} = \sqrt[3]{b-5}$$ Cube both sides

$$a - \cancel{5} = b - \cancel{5}$$
$$+\cancel{5} \qquad +\cancel{5}$$

$$a = b$$ ✓

Therefore, $f(x)$ is one - one.

b) Find f^{-1}, for $f(x) = 2\sqrt[3]{x-5} + 1$

① Write as $y = 2\sqrt[3]{x-5} + 1$

② Switch x & y:

$$x = 2\sqrt[3]{y-5} + 1$$

③ $\dfrac{x-1}{2} = \sqrt[3]{y-5}$

④ $\left(\dfrac{x-1}{2}\right)^3 = y-5$

⑤ $y = \left(\dfrac{x-1}{2}\right)^3 + 5$

⑥ $f^{-1}(x) = \left(\dfrac{x-1}{2}\right)^3 + 5$ (Have to show this last notation for full credit)

#10. $f(x) = \dfrac{8}{x^2-4}$, $g(x) = \sqrt{6-2x}$

a) $(f \circ g)(x) = f(g(x)) = f(\sqrt{6-2x})$

$$= \frac{8}{(\sqrt{6-2x})^2 - 4}$$

Sufficient final answer. Also, best bet to leave unsimplified in order to find dom $f \circ g$.

b) Dom $f \circ g$ = ?

Here we need the denominator of $f \circ g$ to be nonzero and the radicand $6-2x$ to be nonnegative. It's an intersection of these solutions.

- $(\sqrt{6-2x})^2 - 4 \neq 0$

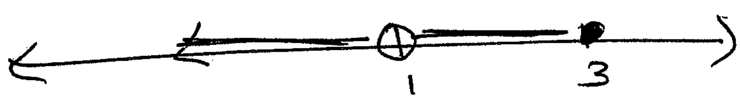
$$6 - 2x - 4 \neq 0$$

$$2x \neq 2$$

$$\boxed{x \neq 1}$$

- $6 - 2x \geq 0$

$$\boxed{x \leq 3}$$



$(-\infty, 1) \cup (1, 3]$

Ans.

10 c) Is f even, odd, neither? Why?

Go straight to $f(-x)$ and see what happens

$$\underbrace{f(-x) = \frac{8}{(-x)^2-4} = \frac{8}{x^2-4} = f(x)}_{f(-x) = f(x)}$$

So f is even.

[Note: People lost pt for not showing that $(-x)^2$ becomes x^2]

d) Is g even, odd, neither? Why?

Go for $g(-x)$ and see what happens.

$$g(-x) = \sqrt{6-2(-x)} = \sqrt{6+2x} \neq g(x)$$ (so, not even)

nor does $\sqrt{6+2x} = g(\cancel{-x})$ (so, not odd)

Hence, g is neither even nor odd.