Math 220 - Quiz (take-home) - Integration by Parts

#1. $\int 2xe^{-x}\,dx = 2\int xe^{-x}\,dx$ $\quad u = x \quad dv = e^{-x}\,dx$

$du = dx \quad v = -e^{-x}$

$2\int xe^{-x}\,dx = 2(-xe^{-x}) - 2\int -e^{-x}\,dx$

This is an $\int e^{w}\,dw$ form since $w = -x$, $dw = -dx$

$= -2xe^{-x} - 2e^{-x} + C$

$= \boxed{-e^{-x}(x+1) + C}$

#2. $\int \ln 6x\,dx$ $\quad u = \ln 6x \quad dv = dx$

$du = \dfrac{dx}{x} \quad v = x$

because by chain rule, $\dfrac{d}{dx}(\ln 6x) = \dfrac{1}{\cancel{6}x}\cdot\cancel{6}$

$\int \ln 6x\,dx = x\ln 6x - \int \dfrac{\cancel{x}\,dx}{\cancel{x}} = \boxed{x\ln|6x| - x + C}$

In general: $\int \ln ax\,dx = x\ln ax - x + C$

#3. $\int x\sqrt{x+1}\,dx$ $\quad$ Method 1: u-sub: $u = x+1$, $x = u-1$

$\dfrac{du}{dx} = 1$, $dx = du$

substitute for both factors

$\int (u-1)(u^{1/2})\,du = \int u^{3/2} - u^{1/2}\,du = \dfrac{2u^{5/2}}{5} - \dfrac{2u^{3/2}}{3} + C$

$= \boxed{\dfrac{2(x+1)^{5/2}}{5} - \dfrac{2(x+1)^{3/2}}{3} + C}$

Method 2: Int. by parts: $u = x \quad dv = (x+1)^{1/2}\,dx$

$du = dx \quad v = \dfrac{2(x+1)^{3/2}}{3}$

Why is $\int dv = \int (x+1)^{1/2}\,dx = \dfrac{2}{3}(x+1)^{3/2}$?

Because $\int (x+1)^{1/2}\,dx = \int w^{1/2}\,dw$, where $w = x+1$

So $\int x\sqrt{x+1}\,dx = \frac{2}{3}x(x+1)^{3/2} - \frac{2}{3}\int (x+1)^{3/2}\,dx$

Again, $\int (x+1)^{3/2}\,dx$ is a form $\int w^{3/2}\,dw$, since, if $w = x+1$, $dw = dx$

The reason we avoid calling it $\int u^n\,du$ is b/c we used "u" in the designation of another term for $\int u\,dv$.

Finally: $\int x\sqrt{x+1}\,dx = \boxed{\frac{2}{3}x(x+1)^{3/2} + \frac{2}{3}\cdot\frac{2}{5}(x+1)^{5/2} + C}$

How does this compare to the u-sub answer?
First of all, it's a lot longer to work out. Second, it doesn't look the same in the final answer.

u-sub: $\frac{2}{5}(x+1)^{5/2} + \frac{2}{3}(x+1)^{3/2} + C_1$ | IBP: $\frac{2}{3}x(x+1)^{3/2} + \frac{4}{15}(x+1)^{5/2} + C_2$

By some crazy trick of algebra I can show these are =.

But don't bother. Always do types like $\int x\sqrt{x+1}\,dx$ as u-sub.

#4. $\displaystyle\int_1^4 2xe^{-x}\,dx = \underbrace{-2xe^{-x}\Big|_1^4 - \int_1^4 -e^{-x}\,dx}_{\text{from \#1 } uv - \int v\,du}$

but with limits inserted $\Longrightarrow uv\Big|_1^4 - \displaystyle\int_1^4 v\,du$

It's best to leave -2 coeff out of the calculation:

$$\rightarrow \; -2\left(xe^{-x}\right)\Big|_1^4 - 2e^{-x}\Big|_1^4$$

$$= -2\left(4e^{-4} - e^{-1}\right) - 2\left(e^{-4} - e^{-1}\right)$$

$$= -8e^{-4} + 2e^{-1} - 2e^{-4} + 2e^{-1} = \boxed{-10e^{-4} + 4e^{-1}}$$

#5. $\displaystyle\int_1^4 \ln 6x\,dx = x\ln 6x\Big|_1^4 - \int_1^4 dx = x\ln 6x\Big|_1^4 - x\Big|_1^4$

$$= 4\ln 24 - \ln 6 - 4 + 1 = \boxed{4\ln 24 - \ln 6 - 3}$$

#6. Find net dist from $t=0$ to 3 sec for object moving at velocity $v(t) = -9.8t + 19.6$ m/sec

$$s(t) = \int_{t_1}^{t_2} v(t)\,dt = \int_0^3 \left(-9.8t + 19.6\right)dt$$

$$= -\frac{9.8t^2}{2} + 19.6t\Big|_0^3 = -4.9t^2 + 19.6t\Big|_0^3$$

$$= -4.9(3^2) + 19.6(3) + \cancel{4.9(0)} - \cancel{19.6(0)}$$

$$= -44.1 + 58.8 = \boxed{14.7\text{ m}}$$

Notice the displacement (distance) fcn. $s(t) = -4.9t^2 + 19.6t + C$ is a parabola. At $t=0$, $s(0) = 0$, since the object has not started moving. So, solving for C:

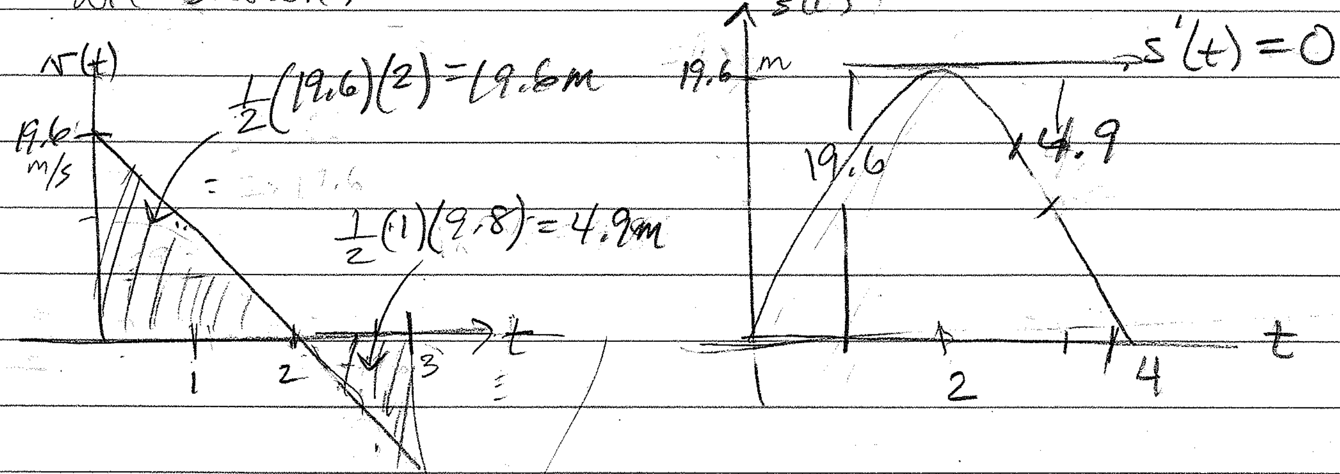
$$s(0) = 0 = -4.9\cancel{(0^2)} + 19.6\cancel{(0)} + C = 0$$

$$C = 0$$

Hence $s(t) = -4.9t^2 + 19.6t$, a parabola that has max at $s'(t) = -9.8t + 19.6 = 0$, or $t = 2$ sec.

This is the velocity fcn. we started with, and it is the velocity of free fall. It represents an object

That is thrown at initial velocity of 19.6 m/sec, which reaches its maximum height at t = 2 sec and then descends. The graphs of velocity + distance are shown:



"Signed area" = Not dist.

$= 19.6 - 4.9\,m = 14.7\,m$