# 6.5 Solving Exponential Equations

## Essential Question

How can you solve an exponential equation graphically?

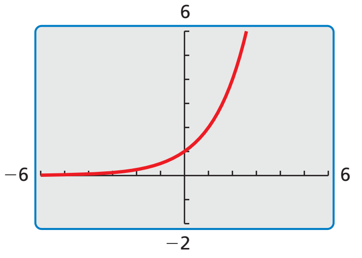
### EXPLORATION 1 Solving an Exponential Equation Graphically

**Work with a partner.** Use a graphing calculator to solve the exponential equation $2.5^{x-3} = 6.25$ graphically. Describe your process and explain how you determined the solution.

### EXPLORATION 2 The Number of Solutions of an Exponential Equation

**Work with a partner.**

**a.** Use a graphing calculator to graph the equation $y = 2^x$.




**b.** In the same viewing window, graph a linear equation (if possible) that does not intersect the graph of $y = 2^x$.

**c.** In the same viewing window, graph a linear equation (if possible) that intersects the graph of $y = 2^x$ in more than one point.

**d.** Is it possible for an exponential equation to have no solution? more than one solution? Explain your reasoning.

**USING APPROPRIATE TOOLS**

To be proficient in math, you need to use technological tools to explore and deepen your understanding of concepts.

### EXPLORATION 3 Solving Exponential Equations Graphically

**Work with a partner.** Use a graphing calculator to solve each equation.

**a.** $2^x = \frac{1}{2}$ **b.** $2^{x+1} = 0$ **c.** $2^x = \sqrt{2}$

**d.** $3^x = 9$ **e.** $3^{x-1} = 0$ **f.** $4^{2x} = 2$

**g.** $2^{x/2} = \frac{1}{4}$ **h.** $3^{x+2} = \frac{1}{9}$ **i.** $2^{x-2} = \frac{3}{2}x - 2$

## Communicate Your Answer

**4.** How can you solve an exponential equation graphically?

**5.** A population of 30 mice is expected to double each year. The number $p$ of mice in the population each year is given by $p = 30(2^n)$. In how many years will there be 960 mice in the population?

# 6.5 Lesson

## Core Vocabulary

exponential equation, *p. 326*

## What You Will Learn

- Solve exponential equations with the same base.
- Solve exponential equations with unlike bases.
- Solve exponential equations by graphing.

## Solving Exponential Equations with the Same Base

**Exponential equations** are equations in which variable expressions occur as exponents.

## Core Concept

### Property of Equality for Exponential Equations

**Words** Two powers with the *same positive base b*, where $b \neq 1$, are equal if and only if their exponents are equal.

**Numbers** If $2^x = 2^5$, then $x = 5$. If $x = 5$, then $2^x = 2^5$.

**Algebra** If $b > 0$ and $b \neq 1$, then $b^x = b^y$ if and only if $x = y$.

### EXAMPLE 1 Solving Exponential Equations with the Same Base

Solve each equation.

**a.** $3^{x+1} = 3^5$ **b.** $6 = 6^{2x-3}$ **c.** $10^{3x} = 10^{2x+3}$

**SOLUTION**

**a.**

| | |
|---|---|
| $3^{x+1} = 3^5$ | Write the equation. |
| $x + 1 = 5$ | Equate the exponents. |
| $-1 \quad -1$ | Subtract 1 from each side. |
| $x = 4$ | Simplify. |

**b.**

| | |
|---|---|
| $6 = 6^{2x-3}$ | Write the equation. |
| $1 = 2x - 3$ | Equate the exponents. |
| $+3 \quad +3$ | Add 3 to each side. |
| $4 = 2x$ | Simplify. |
| $\frac{4}{2} = \frac{2x}{2}$ | Divide each side by 2. |
| $2 = x$ | Simplify. |

**Check**

$$6 = 6^{2x-3}$$
$$6 \stackrel{?}{=} 6^{2(2)-3}$$
$$6 = 6 \checkmark$$

**c.**

| | |
|---|---|
| $10^{3x} = 10^{2x+3}$ | Write the equation. |
| $3x = 2x + 3$ | Equate the exponents. |
| $-2x \quad -2x$ | Subtract $2x$ from each side. |
| $x = 3$ | Simplify. |

## Monitoring Progress

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**Solve the equation. Check your solution.**

**1.** $2^{2x} = 2^6$ **2.** $5^{2x} = 5^{x+1}$ **3.** $7^{3x+5} = 7^{x+1}$

## Solving Exponential Equations with Unlike Bases

To solve some exponential equations, you must first rewrite each side of the equation using the same base.

### EXAMPLE 2 Solving Exponential Equations with Unlike Bases

Solve (a) $5^x = 125$, (b) $4^x = 2^{x-3}$, and (c) $9^{x+2} = 27^x$.

**SOLUTION**

**a.**

| | |
|---|---|
| $5^x = 125$ | Write the equation. |
| $5^x = 5^3$ | Rewrite 125 as $5^3$. |
| $x = 3$ | Equate the exponents. |

**b.**

| | |
|---|---|
| $4^x = 2^{x-3}$ | Write the equation. |
| $(2^2)^x = 2^{x-3}$ | Rewrite 4 as $2^2$. |
| $2^{2x} = 2^{x-3}$ | Power of a Power Property |
| $2x = x - 3$ | Equate the exponents. |
| $x = -3$ | Solve for $x$. |

**Check**

$$4^x = 2^{x-3}$$

$$4^{-3} \stackrel{?}{=} 2^{-3-3}$$

$$\frac{1}{64} = \frac{1}{64} \ \checkmark$$

**c.**

| | |
|---|---|
| $9^{x+2} = 27^x$ | Write the equation. |
| $(3^2)^{x+2} = (3^3)^x$ | Rewrite 9 as $3^2$ and 27 as $3^3$. |
| $3^{2x+4} = 3^{3x}$ | Power of a Power Property |
| $2x + 4 = 3x$ | Equate the exponents. |
| $4 = x$ | Solve for $x$. |

**Check**

$$9^{x+2} = 27^x$$

$$9^{4+2} \stackrel{?}{=} 27^4$$

$$531{,}441 = 531{,}441 \ \checkmark$$

### EXAMPLE 3 Solving Exponential Equations When $0 < b < 1$

Solve (a) $\left(\frac{1}{2}\right)^x = 4$ and (b) $4^{x+1} = \frac{1}{64}$.

**SOLUTION**

**a.**

| | |
|---|---|
| $\left(\frac{1}{2}\right)^x = 4$ | Write the equation. |
| $(2^{-1})^x = 2^2$ | Rewrite $\frac{1}{2}$ as $2^{-1}$ and 4 as $2^2$. |
| $2^{-x} = 2^2$ | Power of a Power Property |
| $-x = 2$ | Equate the exponents. |
| $x = -2$ | Solve for $x$. |

**b.**

| | |
|---|---|
| $4^{x+1} = \frac{1}{64}$ | Write the equation. |
| $4^{x+1} = \frac{1}{4^3}$ | Rewrite 64 as $4^3$. |
| $4^{x+1} = 4^{-3}$ | Definition of negative exponent |
| $x + 1 = -3$ | Equate the exponents. |
| $x = -4$ | Solve for $x$. |

**Check**

$$4^{x+1} = \frac{1}{64}$$

$$4^{-4+1} \stackrel{?}{=} \frac{1}{64}$$

$$\frac{1}{64} = \frac{1}{64} \ \checkmark$$

### Monitoring Progress

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**Solve the equation. Check your solution.**

**4.** $4^x = 256$ **5.** $9^{2x} = 3^{x-6}$ **6.** $4^{3x} = 8^{x+1}$ **7.** $\left(\frac{1}{3}\right)^{x-1} = 27$

## Solving Exponential Equations by Graphing

Sometimes, it is impossible to rewrite each side of an exponential equation using the same base. You can solve these types of equations by graphing each side and finding the point(s) of intersection. Exponential equations can have no solution, one solution, or more than one solution depending on the number of points of intersection.

### EXAMPLE 4 Solving Exponential Equations by Graphing

Use a graphing calculator to solve (a) $\left(\frac{1}{2}\right)^{x-1} = 7$ and (b) $3^{x+2} = x + 1$.

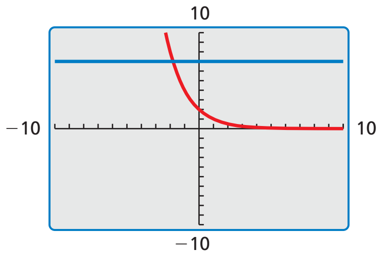
**SOLUTION**

**a. Step 1** Write a system of equations using each side of the equation.

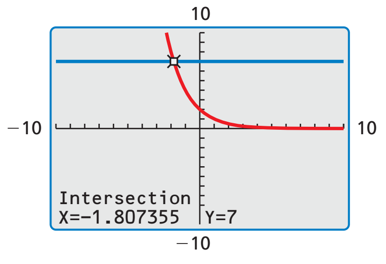
$y = \left(\frac{1}{2}\right)^{x-1}$ Equation 1

$y = 7$ Equation 2

**Step 2** Enter the equations into a calculator. Then graph the equations in a viewing window that shows where the graphs could intersect.




**Step 3** Use the *intersect* feature to find the point of intersection. The graphs intersect at about $(-1.81, 7)$.




▶ So, the solution is $x \approx -1.81$.

**Check**

$\left(\frac{1}{2}\right)^{x-1} = 7$

$\left(\frac{1}{2}\right)^{-1.81-1} \stackrel{?}{=} 7$

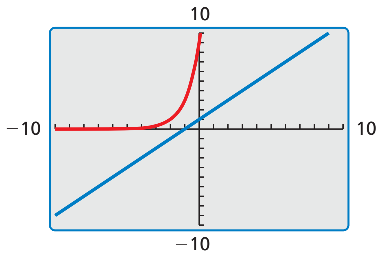
$7.01 \approx 7$ ✓

**b. Step 1** Write a system of equations using each side of the equation.

$y = 3^{x+2}$ Equation 1

$y = x + 1$ Equation 2

**Step 2** Enter the equations into a calculator. Then graph the equations in a viewing window that shows where the graphs could intersect.




▶ The graphs do not intersect. So, the equation has no solution.

### Monitoring Progress

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**Use a graphing calculator to solve the equation.**

**8.** $2^x = 1.8$ **9.** $4^{x-3} = x + 2$ **10.** $\left(\frac{1}{4}\right)^x = -2x - 3$

# 6.5 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

## Vocabulary and Core Concept Check

1. **WRITING** Describe how to solve an exponential equation with unlike bases.

2. **WHICH ONE DOESN'T BELONG?** Which equation does *not* belong with the other three? Explain your reasoning.

| | | | |
|---|---|---|---|
| $2^x = 4^{x+6}$ | $5^{3x+8} = 5^{2x}$ | $3^4 = x + 4^2$ | $2^{x-7} = 2^7$ |

## Monitoring Progress and Modeling with Mathematics

**In Exercises 3–12, solve the equation. Check your solution.** *(See Examples 1 and 2.)*

3. $4^{5x} = 4^{10}$
4. $7^{x-4} = 7^8$
5. $3^{9x} = 3^{7x+8}$
6. $2^{4x} = 2^{x+9}$
7. $2^x = 64$
8. $3^x = 243$
9. $7^{x-5} = 49^x$
10. $216^x = 6^{x+10}$
11. $64^{2x+4} = 16^{5x}$
12. $27^x = 9^{x-2}$

**In Exercises 13–18, solve the equation. Check your solution.** *(See Example 3.)*

13. $\left(\frac{1}{5}\right)^x = 125$
14. $\left(\frac{1}{4}\right)^x = 256$
15. $\frac{1}{128} = 2^{5x+3}$
16. $3^{4x-9} = \frac{1}{243}$
17. $36^{-3x+3} = \left(\frac{1}{216}\right)^{x+1}$
18. $\left(\frac{1}{27}\right)^{4-x} = 9^{2x-1}$

**ERROR ANALYSIS In Exercises 19 and 20, describe and correct the error in solving the exponential equation.**

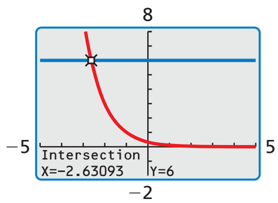
19. ✗
$$5^{3x+2} = 25^{x-8}$$
$$3x + 2 = x - 8$$
$$x = -5$$

20. ✗
$$\left(\frac{1}{8}\right)^{5x} = 32^{x+8}$$
$$(2^3)^{5x} = (2^5)^{x+8}$$
$$2^{15x} = 2^{5x+40}$$
$$15x = 5x + 40$$
$$x = 4$$

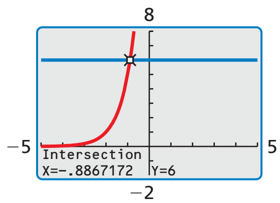
**In Exercises 21–24, match the equation with the graph that can be used to solve it. Then solve the equation.**

21. $2^x = 6$
22. $4^{2x-5} = 6$
23. $5^{x+2} = 6$
24. $3^{-x-1} = 6$

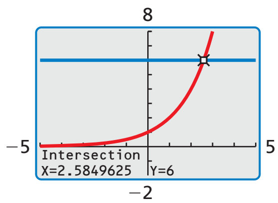
**A.**



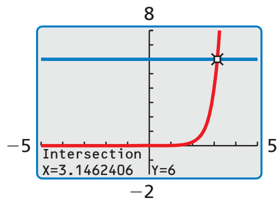

**B.**




**C.**




**D.**




**In Exercises 25–36, use a graphing calculator to solve the equation.** *(See Example 4.)*

25. $6^{x+2} = 12$
26. $5^{x-4} = 8$
27. $\left(\frac{1}{2}\right)^{7x+1} = -9$
28. $\left(\frac{1}{3}\right)^{x+3} = 10$
29. $2^{x+6} = 2x + 15$
30. $3x - 2 = 5^{x-1}$
31. $\frac{1}{2}x - 1 = \left(\frac{1}{3}\right)^{2x-1}$
32. $2^{-x+1} = -\frac{3}{4}x + 3$
33. $5^x = -4^{-x+4}$
34. $7^{x-2} = 2^{-x}$
35. $2^{-x-3} = 3^{x+1}$
36. $5^{-2x+3} = -6^{x+5}$

**In Exercises 37–40, solve the equation by using the Property of Equality for Exponential Equations.**

**37.** $30 \cdot 5^{x+3} = 150$

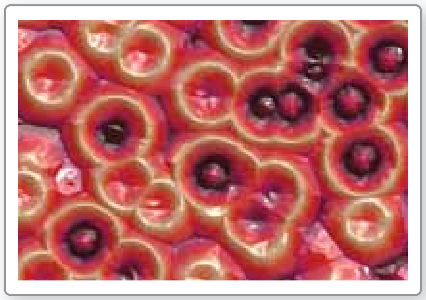
**38.** $12 \cdot 2^{x-7} = 24$

**39.** $4(3^{-2x-4}) = 36$

**40.** $2(4^{2x+1}) = 128$

**41. MODELING WITH MATHEMATICS** You scan a photo into a computer at four times its original size. You continue to increase its size repeatedly by 100% using the computer. The new size of the photo $y$ in comparison to its original size after $x$ enlargements on the computer is represented by $y = 2^{x+2}$. How many times must the photo be enlarged on the computer so the new photo is 32 times the original size?

**42. MODELING WITH MATHEMATICS** A bacterial culture quadruples in size every hour. You begin observing the number of bacteria 3 hours after the culture is prepared. The amount $y$ of bacteria $x$ hours after the culture is prepared is represented by $y = 192(4^{x-3})$. When will there be 200,000 bacteria?



**In Exercises 43–46, solve the equation.**

**43.** $3^{3x+6} = 27^{x+2}$

**44.** $3^{4x+3} = 81^{x}$

**45.** $4^{x+3} = 2^{2(x+1)}$

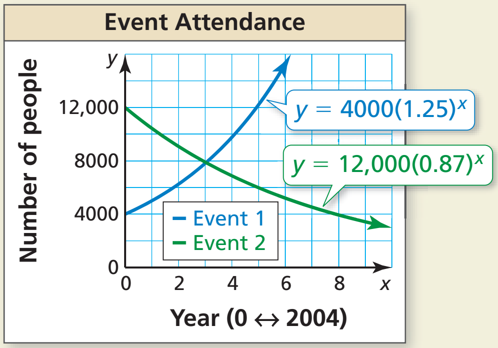
**46.** $5^{8(x-1)} = 625^{2x-2}$

**47. NUMBER SENSE** Explain how you can use mental math to solve the equation $8^{x-4} = 1$.

**48. PROBLEM SOLVING** There are a total of 128 teams at the start of a citywide 3-on-3 basketball tournament. Half the teams are eliminated after each round. Write and solve an exponential equation to determine after which round there are 16 teams left.

**49. PROBLEM SOLVING** You deposit \$500 in a savings account that earns 6% annual interest compounded yearly. Write and solve an exponential equation to determine when the balance of the account will be \$800.

**50. HOW DO YOU SEE IT?** The graph shows the annual attendance at two different events. Each event began in 2004.




a. Estimate when the events will have about the same attendance.

b. Explain how you can verify your answer in part (a).

**51. REASONING** Explain why the Property of Equality for Exponential Equations does not work when $b = 1$. Give an example to justify your answer.

**52. THOUGHT PROVOKING** Is it possible for an exponential equation to have two different solutions? If not, explain your reasoning. If so, give an example.

**USING STRUCTURE In Exercises 53–58, solve the equation.**

**53.** $8^{x-2} = \sqrt{8}$

**54.** $\sqrt{5} = 5^{x+4}$

**55.** $\left(\sqrt[5]{7}\right)^{x} = 7^{2x+3}$

**56.** $12^{2x-1} = \left(\sqrt[4]{12}\right)^{x}$

**57.** $\left(\sqrt[3]{6}\right)^{2x} = \left(\sqrt{6}\right)^{x+6}$

**58.** $\left(\sqrt[5]{3}\right)^{5x-10} = \left(\sqrt[8]{3}\right)^{4x}$

**59. MAKING AN ARGUMENT** Consider the equation $\left(\frac{1}{a}\right)^{x} = b$, where $a > 1$ and $b > 1$. Your friend says the value of $x$ will always be negative. Is your friend correct? Explain.

## Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

**Determine whether the sequence is arithmetic. If so, find the common difference.** *(Section 4.6)*

**60.** $-20, -26, -32, -38, \ldots$

**61.** $9, 18, 36, 72, \ldots$

**62.** $-5, -8, -12, -17, \ldots$

**63.** $10, 20, 30, 40, \ldots$