4.4 Properties of Logarithms; Solving Exponential/Logarithmic Equations

Fundamental Properties of Logarithms (for any base b, and $b \neq 1$):

1.

2.

3.

4.

EX: Evaluate.

$\log_3 1 =$

$\ln 1 =$

$\log_7 7 =$

$\log_{10} 10 =$

$10^{\log x} =$

$4^{\log_4 (x+3)} =$

$\log_6 6^x =$

$\log_9 9^{2x} =$

$e^{\ln 5} =$

$e^{2\ln 5} =$

$3^{5\log_3 10} =$

Properties of Logarithms (for positive real numbers M, N, and $b \neq 1$, and any real number p):

1.

2.

3.

EX: Write the single logarithm in a form with no logarithm of a product, quotient, root, or power, so that you remain with a sum, difference, and/or constant multiple of logarithms. Assume all variables are positive. *Simplify answer accordingly*.

a. $\log\left(\dfrac{100(w+1)^5}{\sqrt[9]{r^4}}\right) =$

b. $\ln\sqrt{\dfrac{x^5}{y^2}} =$

EX: Write each statement as a single logarithm. *Simplify answer accordingly*.

a. $\log_3 18 + 2\log_3 5 - \log_3 50$

b. $5\log_4 x + 2\log_4 5 - \log_4 y - 3\log_4 z$

Change-of-Base Formula (for positive real numbers m, *a*, & *b* with *a*, $b \neq 1$): $\log_b m = \dfrac{\log_a m}{\log_a b}$

$\log_4 30 =$

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EX: Simplify using the change-of-base formula.

a. $(\log_4 7)(\log_7 64)$ b. $(\log_3 e)(\ln 9)$ c. $(\log_2 5)(\log_{25} 16^{-1})$

Exponential and Logarithmic Equations

Solving Exponential Equations

1. Isolate the exponential expression on one side of the equation.
2. Take the log of both sides and "bring down the exponent."
3. Solve for the variable.

EX: Solve.

a. $3^{2x} = 81$.

b. $3^{2x} = 80$

c. $5\left(2^{9x}\right) - 3 = 37$

d. $7^{x} = 4^{2x-1}$

e. $-14 + 3e^{x-4} = 11$

f. $e^{2x} - 4e^{x} = 5$

Solving Logarithmic Equations

1. Write as a single log (or a single log on each side).
2. Write in exponential form or "exponentiate."
3. Solve for the variable. CHECK YOUR SOLUTIONS.

EX: Solve.

a. $\ln x = -3$.

b. $\log_x 625 = 4$

c. $\ln x + \ln(2x+1) = 0$

d. $\log_4 x - \log_4(x-1) = \frac{1}{2}$

e. $\log(x+12) - \log x = \log(x+2)$

f. $\ln(x-1) + \ln(x-3) = 2\ln x$