Add to the formulas from yesterday the following:

- Power reduction formulas: (p. 275)

$$\sin^2 x = \frac{1-\cos 2x}{2} \qquad \left(\tan^2 x = \frac{\sin^2 x}{\cos^2 x}\right)$$

$$\cos^2 x = \frac{1+\cos 2x}{2}$$

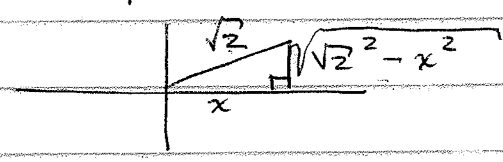
- Half angle formulas (p. 276)

$$\sin\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1-\cos^2 x}{2}}$$

$$\cos\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1+\cos^2 x}{2}}$$

Sec 8.8 con'd

#9 Given $\cos\theta = \frac{x}{\sqrt{2}}$ express $\sin(2\theta)$ + $\cos(2\theta)$ in terms of $\cos\theta$.



$$\sin 2\theta = 2\sin\theta\cos\theta$$
$$= \frac{2\sqrt{2-x^2}}{\sqrt{2}}\cdot\frac{x}{\sqrt{2}}$$
$$= \frac{2x\sqrt{2-x^2}}{2}$$
$$= x\sqrt{2-x^2}$$

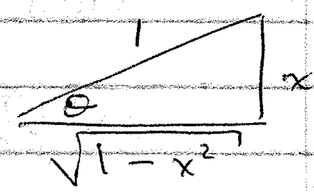
$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$
$$= \cancel{\left(\frac{x}{\sqrt{2}}\right)^2 - \left(\frac{\sqrt{2-x^2}}{\sqrt{2}}\right)^2}$$
$$= \left(\frac{x}{\sqrt{2}}\right)^2 - \left(\frac{\sqrt{2-x^2}}{\sqrt{2}}\right)^2$$
$$= \frac{x^2}{2} - \left(\frac{2-x^2}{2}\right)$$
$$= \frac{2x^2-2}{2} = x^2 - 1$$

#10 Express $\sin(2\sin^{-1}x)$ as a fcn of x with no trig notation:

Hint: Use double angle formula

Noticing that $\sin 2\theta = 2\sin\theta\cos\theta$, we might find this helpful.

$$\sin(\overbrace{2\,\underbrace{\sin^{-1}}_{\text{First}}x}^{\alpha}) \longrightarrow$$



$$\sin(2\sin\theta)$$

So $\cos\theta = \dfrac{\sqrt{1-x^2}}{1}$

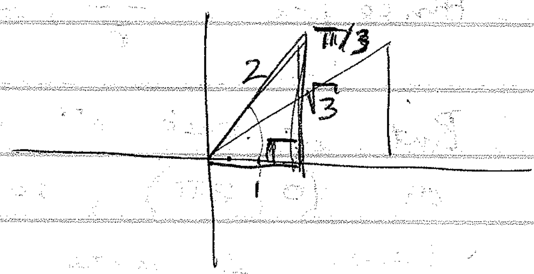
$\sin\theta = x$

Sec 8.9 Solving Trig Equations

Find all solns on $[0, 2\pi)$

#1 a) $\sqrt{3}\csc x - 2 = 0$

$\csc x = \frac{2}{\sqrt{3}} \rightarrow \csc^{-1}\left(\frac{2}{\sqrt{3}}\right) = \frac{\pi}{3}$



$\csc x = \frac{1}{\sin x} = \frac{2}{\sqrt{3}}$

so find x where

$\sin x = \sqrt{3}/2$

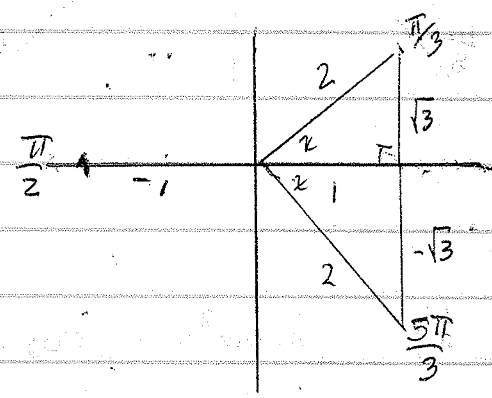
$\boxed{x = \pi/3,\ 2\pi/3}$

b) $2\cos^2 x + \cos x - 1 = 0$

$(2\cos x - 1)(\cos x + 1) = 0$

$\cos x = \frac{1}{2}, \quad \cos x = -1$

$x = \frac{\pi}{3}, \frac{5\pi}{3}, \pi$



c) $-\cot(2x) = 2 + \tan(2x)$

$\frac{-1}{\tan(2x)} = 2 + \tan(2x)$ multiply by $\tan(2x)$

$-1 = 2\tan(2x) + \tan^2(2x)$

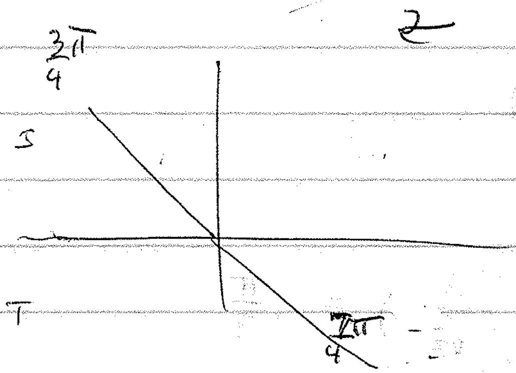
$0 = \tan^2(2x) + 2\tan(2x) + 1$ ← factors as $(a+1)(a+1)$

$0 = \left[\tan(2x) + 1\right]^2$

$\tan(2x) = -1$ — Find what angle $2x$ gives $\tan 2x = -1$

$\cancel{\tan^{-1}(-1) = 2x}$ ~~To isolate x~~

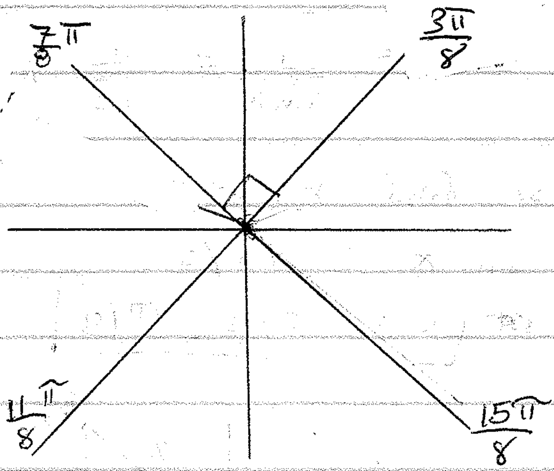
$\cancel{\frac{\tan^{-1}(-1)}{2} = x}$ ~~Now find where~~ $\cancel{\tan^{-}}$



$\tan(2x) = -1$ at

$2x = \frac{3}{4}\pi,\ \frac{7\pi}{4}$

Hence $\boxed{x = \frac{3\pi}{8},\ \frac{7\pi}{8}}$



But we seek all solns on $[0, 2\pi)$, so we have 2 more answers since we cut angle in 2.

You could check to see that each of these gives $\tan(2x) = -1$

#1 d) $2\sin^2 x - \cos 2x = 0$ Try double angle for $\cos 2x$

$2\sin^2 x - (\cos^2 x - \sin^2 x) = 0$

$2\sin^2 x + \sin^2 x - \cos^2 x = 0$

$3\sin^2 x - \cos^2 x = 0$ Now use $\cos^2 x = 1 - \sin^2 x$

$3\sin^2 x - (1 - \sin^2 x) = 0$

$3\sin^2 x + \sin^2 x = 1$

$4\sin^2 x = 1$

$\sin^2 x = \frac{1}{4}$

$\sin x = \pm\frac{1}{2}$ This soln lies in all quadrants.

$x = \frac{\pi}{6},\ \frac{5\pi}{6}$ for $\sin x = \frac{1}{2}$

$x = \frac{7\pi}{6},\ \frac{11\pi}{6}$ for $\sin x = -\frac{1}{2}$

\# 1e $\tan^2(2x) + 5 = 3\sec^2(2x)$ Take advantage

$\sec^2(2x) - 1 + 5 = 3\sec^2(2x)$ of $\tan^2\alpha + 1 = \sec^2\alpha$

$0 = 3\sec^2(2x) - \sec^2(2x) - 4$

$0 = 2\sec^2(2x) - 4$

$0 = \sec^2(2x) - 2$ Algebra

$\sec^2(2x) = 2$

$\sec(2x) = \pm\sqrt{2} \longrightarrow$ Hey! $\sec(2x) = \dfrac{1}{\cos(2x)}$

and we know $\pm 1/\sqrt{2}$

is $\cos(\frac{\pi}{4})$ + $\cos(\frac{7\pi}{4})$

$\sec(2x) = \pm\sqrt{2} \longleftarrow$ + $\cos(\frac{3\pi}{4})$ + $\cos(\frac{5\pi}{4})$

$2x = \dfrac{\pi}{4}, \dfrac{3\pi}{4}, \dfrac{5\pi}{4}, \dfrac{7\pi}{4}$

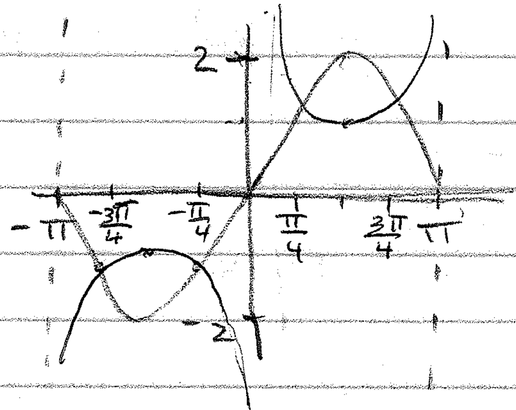
and $\boxed{x = \dfrac{\pi}{8}, \dfrac{3\pi}{8}, \dfrac{5\pi}{8}, \dfrac{7\pi}{8}}$

We still have another "π to go" in $[0, 2\pi)$

Generate the rest by adding $\frac{2\pi}{8}$ successively.

$\boxed{x = \dfrac{9\pi}{8}, \dfrac{11\pi}{8}, \dfrac{13\pi}{8}, \dfrac{15\pi}{8}}$

#2) a) $2\sin x = \csc x$



Before we try to do this "algebraically", consider the graphs of $\csc x$ + $2\sin x$. Where do they intersect.

4 pts of intersection; Do algebra

Given: $2\sin x = \csc x$

Set = zero: $2\sin x - \csc x = 0$

Use def of $\csc x$: $2\sin x - \dfrac{1}{\sin x} = 0$

LCD: $2\sin^2 x - 1 = 0$

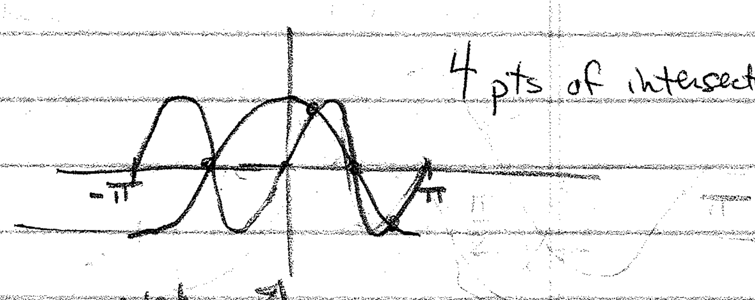
Solve: $2\sin^2 x = 1 \Rightarrow \sin x = \pm\sqrt{2}/2$

$x = \pm\pi/4, \pm 3\pi/4$

c) $\sin x \cos x = \frac{1}{2}\cos x$

$2\sin x \cos x = \cos x$

$\underbrace{\sin 2x = \cos x}$

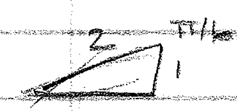


4 pts of intersection

Good for analyzing by graph

but to solve, leave as $\sin x \cos x = \frac{1}{2}\cos x$.

$2\sin x \cos x = \cos x$



$2\sin x \cos x - \cos x = 0$

$\cos x\,(2\sin x - 1) = 0 \implies x = \frac{\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}$

$\cos x = 0, \quad 2\sin x - 1 = 0$

on $[-\pi, \pi]$