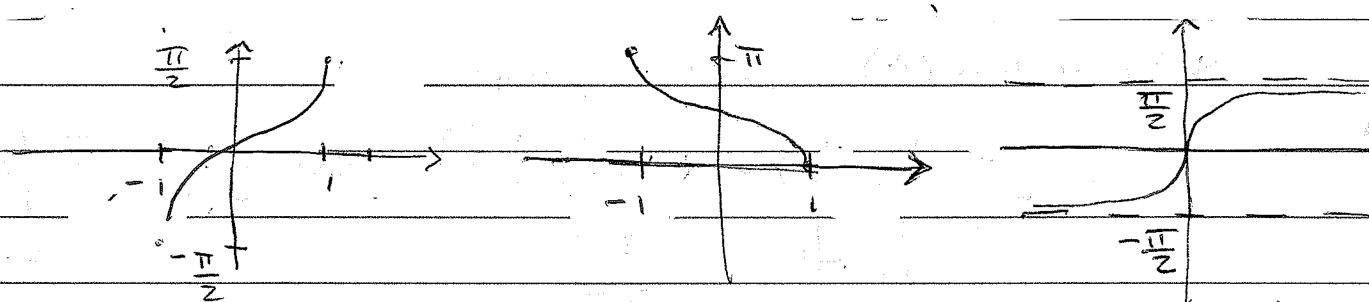
Sec 8.6 Inverse trig fns

- Remember, arcsin x, or $\sin^{-1} x$, is defined on a domain of $[-1, 1]$, and since we restricted $\sin x$ to a domain of $[-\pi/2, \pi/2]$ so we could produce the inverse, the range of $\sin^{-1} x$ is also $[-\pi/2, \pi/2]$.
- The arccos x, or $\cos^{-1} x$, domain is also $[-1, 1]$ but range reflects choice of $[0, \pi]$ for domain restriction on $\cos x$ that produced the inverse.

#1 Graphs



$y = \sin^{-1} x$ Dom $= [-1, 1]$ $y = \cos^{-1} x$ Dom $= [-1, 1]$ $y = \tan^{-1} x$ Dom $= \mathbb{R}$

#2

a) Dom $f(x) = \cos^{-1}(x+2)$, where $-1 \le x+2 \le 1$, or $-3 \le x \le -1$. Note, too, that the fcn $y = (\cos x) - 2$ is the inverse of $f(x) = \cos^{-1}(x+2)$, and the Range y is $[-3, -1]$.

b) $f(x) = \arctan(x^2 + 3x)$; $x^2 + 3x \in \mathbb{R}$
so obviously $x \in \mathbb{R}$, that is Dom f is $(-\infty, \infty)$

c) $f(x) = \arcsin(1-x)$; $-1 \le 1 - x \le 1$

$$-2 \le -x \le 0$$

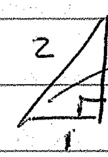
$$2 \ge x \ge 0$$

So Dom f is $[0, 2]$

(#3) a) $\sin^{-1}(-1) = -\frac{\pi}{2}$ (graph)

b) $\arcsin(-\sqrt{2}/2) = -\frac{\pi}{4}$ (graph or table value)

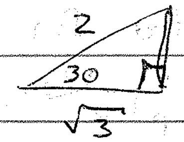
c) $\cos^{-1}(0) = \pi/2$ (graph)

d) $\arccos(-\frac{1}{2}) = \frac{2\pi}{3}$ (graph) $\longrightarrow$ Also 

60°, but in Q2 since $\cos^{-1}$ is neg.

e) $\arcsin(\sqrt{3}/2) = \pi/3$ (table value)

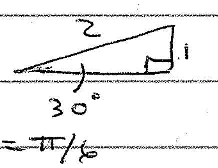
f) $\tan^{-1}(-1) = -\pi/4$ (table value)

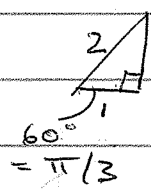
g) $\tan^{-1}(1/\sqrt{3}) = \pi/6 \longleftarrow$ 

or table

h) $\arctan(0) = 0$ graph

i) $\csc[\sin^{-1}(\frac{1}{2}) - \cos^{-1}(\frac{1}{2})] = \csc[\frac{\pi}{6} - \frac{\pi}{3}] = \csc(-\frac{\pi}{6})$





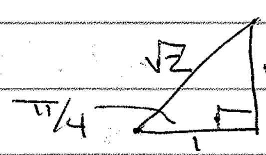
$= -2$

Since $\csc = \frac{1}{\sin}$

(#4) This question asks what inverse fcn will yield the angle values seen.

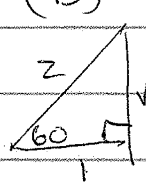
Again, the graph or the trig table is your best source.

(a) $\pi/4 = \arcsin(\sqrt{2}/2) = \arccos(\sqrt{2}/2) = \arctan(1)$



All in Q1 are positive

(b) $-\pi/3$ Look at the graphs, left of zero and from our knowledge of basic trig values (see table), we find



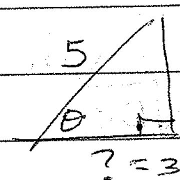
$-\pi/3 = \sin^{-1}(-\sqrt{3}/2) = \cos^{-1}(?)$ No domain value for arccos produces $-\pi/3$

$= \tan^{-1}(-\sqrt{3})$ table

Sec 8.6 con'd

#5) a) $\sin^{-1}\left(\frac{4}{5}\right) = \cos^{-1}(?)$

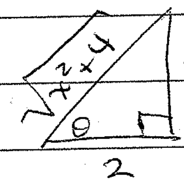
To find ?, we must construct the right triangle that yielded $\sin^{-1}(4/5)$, determine the missing side, + express the cosine ratio from this. Notice we won't need to find the angle itself.



$? = \sqrt{25-4}$
$= \sqrt{9}$
$= 3$

$\cos^{-1}(3/5) = \sin^{-1}(4/5)$

b) $\tan^{-1}(x/2) = \sin^{-1}(?)$



$\sin\theta = \frac{x}{\sqrt{x^2+4}}$

$\tan^{-1}(x/2) = \sin^{-1}\left(x/\sqrt{x^2+4}\right)$

c) $\cos^{-1}(-1) = \pi$, which is not in the range of $\sin^{-1}x$, so $\sin^{-1}(?)$ has no solution.

#4 con'd

c) $\frac{\pi}{2}$ is in range of $\sin^{-1}$, $\cos^{-1}$ but not $\tan^{-1}$

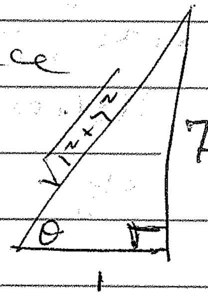
$\frac{\pi}{2} = \sin^{-1}(1) = \cos^{-1}(0)$ are the only two.

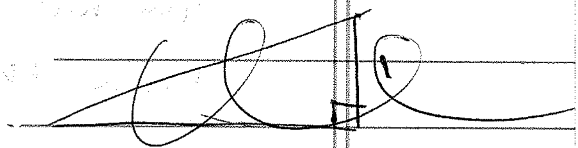
d) 0 is in range of all three:

$0 = \sin^{-1}(0) = \cos^{-1}(1) = \tan^{-1}(0)$

#6) Evaluate w/o a calculator — These require again describing a right triangle with sides implied by the value given. For example,

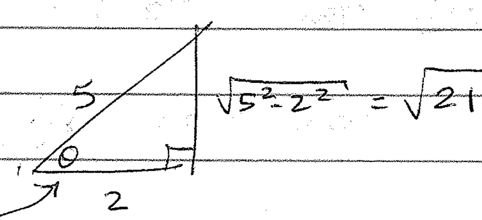
a) ~~$\tan^{-1}(7)$ means a triangle whose~~ tan ratio is $7/1$ would have missing side $\sqrt{50}$, since the triangle → has hypotenuse $\sqrt{1^2+7^2} = \sqrt{50}$





Thus, $\cos(\tan^{-1} 7) = \boxed{1/\sqrt{50}}$

b) $\sin\big(\underbrace{\cos^{-1}\left(\tfrac{2}{5}\right)}_{\theta}\big)$



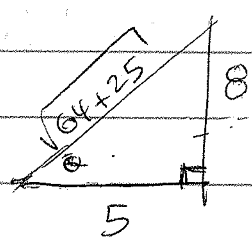
$\boxed{\sin\theta = \sqrt{21}/5}$

c) $\tan^{-1}(\tan(\pi/5)) = \boxed{\pi/5}$ (note, $\pi/5 \in$ Dom $\tan^{-1}$)

d) $\cos^{-1}(\cos(-\pi/4)) = \cos^{-1}(\cos(\pi/4)) = \boxed{\pi/4}$

Not in dom $\cos^{-1}$ but $\cos(-\pi/4)$ is defined → Equivalent angle that is in dom $\cos^{-1}$

e) $\cot\big(\underbrace{\tan^{-1}(8/5)}_{\theta}\big)$



We didn't need the hypotenuse!

$\cot\theta = 5/8$

Notice that dom $\tan^{-1}$ is $\mathbb{R}$

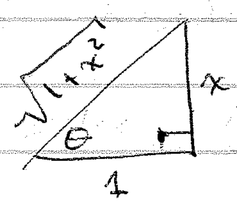
Sec 8.6 con'd

#7 a) $\sin(\arctan x) = \sin(\tan^{-1} x) = ?$

You don't need to rewrite this as I did, but it's more familiar notation, I think.

You need to create the right triangle whose $\tan\theta$ is x, that is, for which

$$\tan\theta = \frac{x}{1} \Rightarrow \tan^{-1}\left(\frac{x}{1}\right) = \theta$$

Thus, 

is the triangle needed

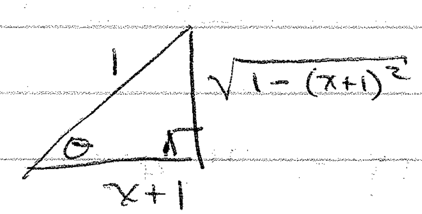
$\sin\theta$ is therefore $x / \sqrt{1+x^2}$

b) $\cos^{-1}(\cos x) = x$ ~~[illegible]~~

for $x \in [0, \pi]$, i.e., for $\cos x \in [-1, 1]$

c) $\tan(\underbrace{\cos^{-1}(x+1)}_{\theta})$

We need the right triangle for which $\cos\theta = \frac{x+1}{1}$, hence $\cos^{-1}\left(\frac{x+1}{1}\right) = \theta$

That is, 

Hence, $\tan\theta = \sqrt{1-(x+1)^2} / x+1 = \sqrt{-x^2-2x} / x+1$

#8) What's wrong with these?

a) $\sin\left(\frac{5\pi}{4}\right) = -\frac{\sqrt{2}}{2}$ so $\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right) = \frac{5\pi}{4}$

Since $\frac{5\pi}{4} \notin$ Restricted domain of sine on which the inverse is taken, then

$\sin^{-1}(\sin x) \neq x$, in this case $\frac{5\pi}{4}$.

b) $\tan^{-1}(\tan 5)) = 5$

Again, the restricted dom of tangent on which $\tan^{-1}$ is defined is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

$5 \notin (-\pi/2, \pi/2)$, since $5 \text{ rad} > \frac{\pi}{2} \text{ rad}$.

c) $\cos^{-1} x = \sec x$

Wrong notation! $\cos^{-1} x \neq \frac{1}{\cos x} = \sec x$

d) $\tan^{-1} x = \frac{\sin^{-1} x}{\cos^{-1} x}$

In general, it is not the case that

$f^{-1} = g^{-1}/h^{-1}$ when $f = g/h$.

For example, $\frac{\sin^{-1}(1)}{\cos^{-1}(1)} =$ undefined since $\frac{\pi/2}{0}$ is undefined

But, $\tan^{-1}(1) = \pi/4$.