Sec 8.2

#1 Similar Δ's are in proportion with respect to lengths of sides.

$$3, 5, 6 \sim 6, 10, 12 \quad \text{for ex.}$$

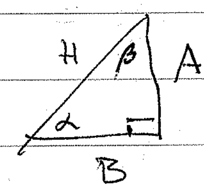
$$\text{or } 3, 5, 6 \sim 3/2, 5/2, 3$$

#2

$$\tan\theta = \frac{opp}{adj} = \frac{opp/hyp}{adj/hyp} = \frac{\sin\theta}{\cos\theta}$$

$$\sec\theta = \frac{hyp}{adj} = \frac{1}{\cos\theta} = \frac{1}{adj/hyp}$$

#3 a) $H = 6$ cm, $A = 2$ cm



Model for all parts of this problem

(A = Altitude
B = Base
H = Hypotenuse)

$$B = \sqrt{H^2 - A^2} = \sqrt{36 - 4} = \sqrt{32} = 4\sqrt{2} \text{ cm}$$

$$\sin\alpha = \frac{A}{H} = \frac{2 \text{ cm}}{6 \text{ cm}} = \frac{1}{3}$$

$$\cos\alpha = \frac{B}{H} = \frac{4\sqrt{2} \text{ cm}}{6 \text{ cm}} = \frac{2\sqrt{2}}{3} \text{ cm}$$

$$\tan\alpha = \frac{A}{B} = \frac{2 \text{cm}}{4\sqrt{2} \text{ cm}} = \frac{1}{2\sqrt{2}} = \frac{\sqrt{2}}{2 \cdot 2} = \frac{\sqrt{2}}{4}$$

The reciprocal trig fcns are:

$$\csc\alpha = \frac{1}{\sin\alpha} = \frac{H}{A} = \frac{6 \text{cm}}{2 \text{cm}} = 3$$

$$\sec\alpha = \frac{1}{\cos\alpha} = \frac{H}{B} = \frac{6 \text{ cm}}{4\sqrt{2} \text{ cm}} = \frac{3}{2\sqrt{2}} = \frac{3\sqrt{2}}{2 \cdot 2} = \frac{3\sqrt{2}}{4}$$

$$\cot\alpha = \frac{1}{\tan\alpha} = \frac{B}{A} = \frac{4\sqrt{2} \text{ cm}}{2 \text{ cm}} = 2\sqrt{2}$$

Do the same for $\sin\beta$, $\cos\beta$, $\tan\beta$ etc., with A + B reversed in the ratios.

#4) ~~cos θ~~ $\cos\theta = \frac{2}{3}$ hyp = 6 in

Find adj + opp sides to θ.

The reduced ratio of $\cos\theta = \frac{2}{3}$, which comes from $\cos\theta = \frac{adj}{hyp} = \frac{adj}{6\text{ in}}$.

Equating the fractions:

$$\frac{2}{3} = \frac{adj}{6} \longrightarrow adj = 4 \text{ in} = B \text{ (base)}$$

Knowing 2 sides + applying Pyth thm:

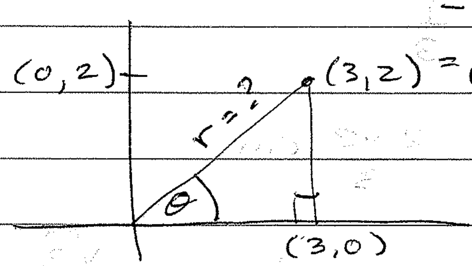
$$A^2 + B^2 = H^2$$

we find $A^2 = H^2 - B^2 = (6^2 - 4^2)\text{ in}^2$

$$A = \sqrt{36-16}\ \text{in}$$

$$= \sqrt{20}\ \text{in} = 2\sqrt{5}\ \text{in}$$

#5)



("Std position" is in QI)

Using the rectangular → trigonometric conversion

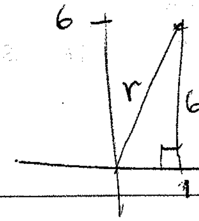
$\cos\theta = \frac{x}{r}$, $\sin\theta = \frac{y}{r}$, $\tan\theta = \frac{y}{x}$, where $\boxed{x = 3,\ y = 2}$

we see we need r.. Using Pyth thm:

$$x^2 + y^2 = r^2$$

$$3^2 + 2^2 = r^2 \longrightarrow r = \sqrt{9+4} = \boxed{\sqrt{13} = r}$$

a) Hence, $\boxed{\cos\theta = \frac{3}{\sqrt{13}}}$, $\boxed{\sin\theta = \frac{2}{\sqrt{13}}}$, $\boxed{\tan\theta = \frac{2}{3}}$

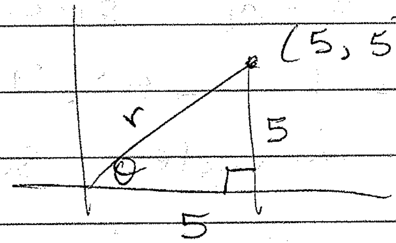
b) 

$(1,6) = (x,y)$

$r^2 = 1^2 + 6^2 = 37$

$\boxed{r = \sqrt{37}}$

$\boxed{\sin\theta = \frac{6}{\sqrt{37}}}$ $\boxed{\cos\theta = \frac{1}{\sqrt{37}}}$ $\boxed{\tan\theta = \frac{6}{1} = 6}$

c) 

$(5,5)$

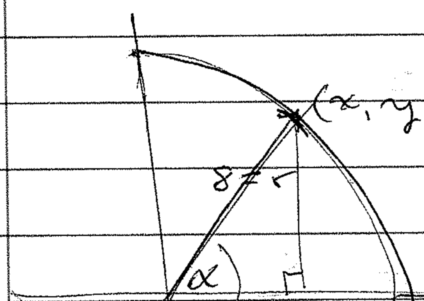
$r^2 = 5^2 + 5^2 = 50$

$r = \sqrt{50} = \boxed{5\sqrt{2} = r}$

(Notice that the ~~taiangle~~ triangle is a 45-45-90, or $1-1-\sqrt{2}$ triangle, scaled up by a factor of 5)

$\boxed{\sin\theta = \frac{5}{5\sqrt{2}} = \frac{1}{\sqrt{2}}}$, $\boxed{\cos\theta = \frac{5}{5\sqrt{2}} = \frac{1}{\sqrt{2}}}$, $\tan\theta = \frac{1/\sqrt{2}}{1/\sqrt{2}}$

$\boxed{\tan\theta = 1}$

#6 

a) $\sin\beta = 0.2 = \frac{y}{8}$

Change 0.2 to $\frac{2}{10} = \frac{1}{5} = \sin\beta$ and solve for y: $\sin\beta = \frac{1}{5} = \frac{y}{8}$

We have $y = 8/5$. Coordinate x is found from Pyth thm $x^2 + y^2 = r^2$:

$$x^2 + \left(\frac{8}{5}\right)^2 = 8^2$$

$$x = \sqrt{64 - 64/5} = \sqrt{\frac{320-64}{5}} = \sqrt{\frac{256}{5}}$$

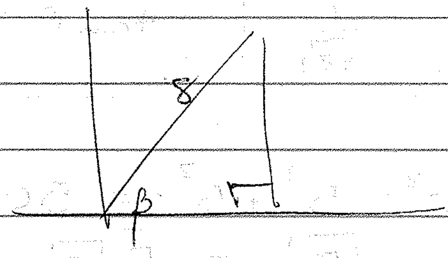
$\boxed{x = 16/\sqrt{5}}$ $\boxed{y = 8/5}$

1.6
1.6
96
16
2.56

64
5
320
-64
256

Note: Answer in book for x is incorrect. If we rationalize denom of $\frac{16}{\sqrt{5}}$ as $\frac{16\sqrt{5}}{5}$ we see the error in the key $\frac{16\sqrt{6}}{5}$.

b)



$\tan\beta = 1$ indicates that $x = y$, since β must be $45°$. This is the $1-1-\sqrt{2}$ right triangle. By similar triangles: $\frac{x}{r} = \frac{x}{8} = \frac{1}{\sqrt{2}} \Rightarrow \boxed{x = 8/\sqrt{2}}$

and $\frac{y}{r} = \frac{y}{8} = \frac{1}{\sqrt{2}} \Rightarrow \boxed{y = 8/\sqrt{2}}$

Rationalizing the denominator of both

$$x = \frac{8}{\sqrt{2}} = \frac{8\sqrt{2}}{2} = 4\sqrt{2}$$

$$y = \frac{8}{\sqrt{2}} = \frac{8\sqrt{2}}{2} = 4\sqrt{2}$$

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c) #7) $\cos\alpha = \frac{3}{7} = \frac{x}{r} = \frac{x}{1}$ ← $r = 1$ since this is on the unit circle

$$\boxed{x = 3/7}$$

Again, by Pyth Thm:

$$y = \sqrt{r^2 - x^2}$$

$$= \sqrt{1^2 - (3/7)^2} = \sqrt{1 - 9/49} = \sqrt{\frac{49-9}{49}}$$

$$y = \frac{\sqrt{40}}{7} = \boxed{\frac{2\sqrt{10}}{7} = y}$$

#7b $\tan\alpha = 1$ is given.

As before, it is known the $\tan\alpha = 1$ indicates a $1-1-\sqrt{2}$ right triangle. Setting up the similar rt. triangle:

$$\frac{x}{r} = \frac{x}{1} = \frac{1}{\sqrt{2}} \Rightarrow \boxed{x = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}}$$

$$\text{and } \boxed{y = \frac{\sqrt{2}}{2}}$$

Alternate approach: $\tan\alpha = y/x = 1$

so $y = x$. Pyth. Thm. gives

$$x^2 + y^2 = 1^2$$

By substitution: $x^2 + x^2 = 2x^2 = 1$
$y = x$

$$x^2 = 1/2 \Rightarrow x = \sqrt{1/2} = \sqrt{2}/2$$

$$y = \sqrt{2}/2$$

#7c) $\tan\alpha = 9/4 = y/x$ The ratio of y/x is $9/4$, but since this is a unit ~~to~~ circle, $r = 1$, so the sides can't be 9 + 4. By ~~Pyth~~ Pyth. Thm., if the triangle were scaled up to $4-9-r$, we'd have:

$$4^2 + 9^2 = r^2 \Rightarrow r^2 = 97$$

So, dividing each term by 97 gives

$$\frac{4^2}{97} + \frac{9^2}{97} = 1 \Rightarrow \boxed{x = \frac{4}{\sqrt{97}},\ y = \frac{9}{\sqrt{97}}}$$

Sec 8.3

#1 Use the trig table with the corresponding θ to evaluate these trig fcns. at the given angle.

a) $\csc 30^\circ = \frac{1}{\sin 30^\circ} = \frac{1}{1/2} = 2$

b) $\cot 45^\circ = \frac{1}{\tan 45^\circ} = \frac{1}{1} = 1$

c) $\sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$ d) $\cos \frac{\pi}{4} = \frac{\sqrt{2}}{2}$

e) $\sec \frac{\pi}{6} = \frac{1}{\cos \pi/6} = \frac{1}{\sqrt{3}/2} = \frac{2}{\sqrt{3}}$

f) $\tan \frac{\pi}{4} = 1$

~~#2 A reference $\measuredangle$ is~~

#2 Angle measure $\theta = \frac{\pi}{4}$ (45°) corresponds to the $1-1-\sqrt{2}$ right triangle. Scaling up to $x-y-6$ gives the proportion

$$\frac{x}{1} = \frac{y}{1} = \frac{6}{\sqrt{2}}$$

Hence $x = y = \frac{6}{\sqrt{2}} = \frac{6\sqrt{2}}{2} = 3\sqrt{2}$

#3 a) $4^2 + (4\sqrt{3})^2 = 16 + 48 = 64 = 8^2$; so this is a right triangle with sides $4 - 4\sqrt{3} - 8$.

b) The other two angles are probably evident in the table, proportioned to $4 - 4\sqrt{3} - 8$.

Dividing by 4 so one side $= 1$, we can determine which triangle this is:

$$4 - 4\sqrt{3} - 8 \longrightarrow 1 - \sqrt{3} - 2$$

The other two angles are therefore 30° & 60°
$(\pi/6$ & $\pi/3)$

#4 $\alpha = 12^\circ$ & $\beta = 67^\circ \Rightarrow \cos(\alpha+\beta) = \cos(12+67)$
$= \cos 79^\circ$

From the trig table, and from Idea 8.3.2 $\left.\right\}$ $\cos 79^\circ = \cos(90 - 11^\circ) = \sin 11^\circ$
$\sin 11^\circ = .1908$

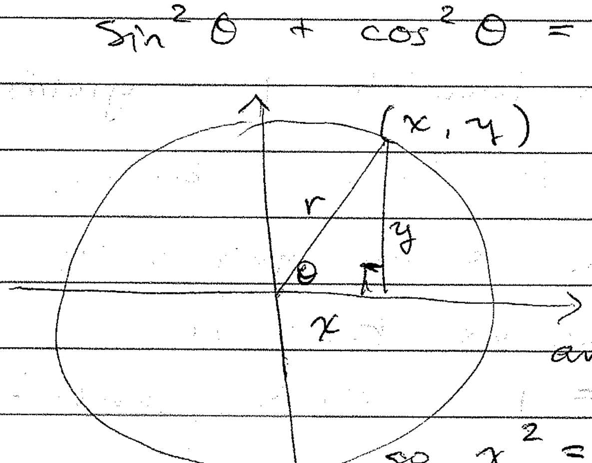
Does this equal $\cos 12^\circ + \cos 67^\circ$?
$\cos 12^\circ = .9781$, $\cos 67^\circ = .3907$
$.9781 + .3907 \neq .1908$

so the example (called a "counterexample") proves $\cos(\alpha+\beta) \neq \cos\alpha + \cos\beta$

#5 Does $\sin(\alpha+\beta) = \sin\alpha + \sin\beta$?
We just need one counterexample (where this is not true) to say it is not true for any or all α, β.

$\sin 45 = .7071$ $\quad$ $\sin 30^\circ = .5000$
$\sin(45 + 30) = \sin 75 = .2588$
$\sin 45 + \sin 30 = .7071 + .5000$
$= 1.2071 \neq .2588$

#6 $\sin^2\theta + \cos^2\theta = 1$ is an important trig identity. It comes from the circle at left, where



$x^2 + y^2 = r^2$

and $\frac{x}{r} = \cos\theta$ $\quad$ $\frac{y}{r} = \sin\theta$

so $x^2 = r^2\cos^2\theta$, $y^2 = r^2\sin^2\theta$

Hence $r^2\cos^2\theta + r^2\sin^2\theta = r^2$ or $\sin^2\theta + \cos^2\theta = 1$

The small trig table $(0, \pi/6, \pi/4, \pi/3, \pi/2, \pi)$ provides many examples of how this identity holds true.

For ex: $\sin \pi/6 = 1/2$ $\quad \cos \pi/6 = \sqrt{3}/2$

$$\sin^2 \pi/6 + \cos^2 \pi/6$$
$$= (\sin \pi/6)^2 + (\cos \pi/6)^2$$
$$= \left(\tfrac{1}{2}\right)^2 + \left(\tfrac{\sqrt{3}}{2}\right)^2$$
$$= \tfrac{1}{4} + \tfrac{3}{4} = 1$$

For angles other than these well known ones, the identity still holds true, but since the larger table gives decimal approximations, the sum might be ≈ 1.

For ex., $\theta = 28°$, $\sin \theta = .4695$, $\cos \theta = .8829$

and so $\sin^2 \theta + \cos^2 \theta = (.4695)^2 + (.8829)^2$

$=$

#7 Evaluate $\sin^2 50° + \sin^2 40°$ w/o calculator.

You probably thought of $\sin^2 \theta + \cos^2 \theta = 1$, but this is $\sin^2 \theta + \sin^2 \beta$ (2 sines, different ∡).

However, $50 + 40 = 90$, so we might have some luck with the phase shift of $\sin \to \cos$.

$$\sin \theta = \cos(90 - \theta) \text{ in quad I.}$$

(e.g. $\sin 30 = \cos 60$
$\sin 45 = \cos 45$
$\sin 17 = \cos 63$
etc)

Thus, $\sin 40 = \cos(90-40) = \cos 50$

So $\sin^2 50^\circ + \sin^2 40^\circ$

$= (\sin 50^\circ)^2 + (\sin 40^\circ)^2$

$= (\sin 50^\circ)^2 + (\cos 50^\circ)^2$

$= \sin^2 50^\circ + \cos^2 50^\circ$

$= 1$ //

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8 or 9 in ~~Sec~~ 8.2 (I think) asked how $\sin\theta$ + $\cos\theta$ compare in magnitude on $[0, \pi/2]$. We know that $\sin\theta$ lies between 0 + $\sqrt{2}/2$ on $[0, \pi/4]$ + between $\sqrt{2}/2$ + 1 on $[\pi/4, \pi/2]$. Cos θ goes the other way, so:

$$0 \le \sin\theta \le \cos\theta \le \sqrt{2}/2 \quad \text{on } [0, \pi/4]$$

and $$\sqrt{2}/2 \le \cos\theta \le \sin\theta \le 1 \quad \text{on } [\pi/4, 1]$$