Sec 7.1 Inequality axioms, $a, b \in \mathbb{R}$

- If $a < b$ then $a \pm c < b \pm c$
- If $a < b$ then $\frac{a}{c} < \frac{b}{c}$ and $ac < bc$

as long as $c > 0$ (i.e. c is positive)

If $c < 0$ (negative), then $a < b \rightarrow ac > bc$

and $a < b \rightarrow \frac{a}{c} > \frac{b}{c}$

switches for $c<0$ as a multiplier

Ex1 $a = 3$, $b = 6$, $c = 1 > 0$

$\boxed{c > 0}$

$3 < 6$, $3 + 1 < 6 + 1$, $3 - 1 < 6 - 1$

$4 < 7$ ✓ $2 < 5$ ✓

✓ $\frac{3}{1} < \frac{6}{1}$, $3 \cdot 1 < 6 \cdot 1$

$3 < 6$ ✓

Ex $a = 3$, $b = 6$, $c = -1 < 0$

$\boxed{c < 0}$

$3 < 6$, $3 + (-1) < 6 + (-1)$, $3 - (-1) < 6 - (-1)$

$2 < 5$ ✓ $4 < 7$ ✓

$3 < 6 \rightarrow 3(-1) \overset{?}{>} 6(-1)$

$-3 > -6$

$-6 < -3$

-6 -3 0

$3 < 6 \rightarrow \frac{3}{-1} \overset{?}{>} \frac{6}{-1} \longrightarrow$ ~~Blem~~ $-3 > -6$

yes, indeed

From Symmetry go to $\Longrightarrow$ Transitive property $a < b, b < c \rightarrow a < c$

$a \square b$, $b \square c$,

$\Longrightarrow a \square c$

Where $\square$ is $=, >, <$

What about $\geq, \leq$ for everything ~~about~~ above?

"generic"
Inequality $\leq, \geq$ less (greater) than or equal to

Strict inequality $<, >$

(Sometimes you'll see $\leqq, \geqq$)

Finally, if $a < b$, and a, b both > 0
or both < 0

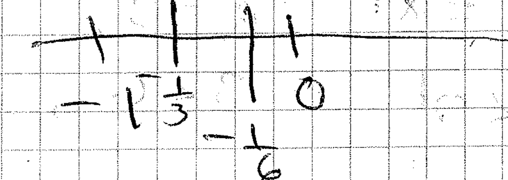
then $\frac{1}{a} > \frac{1}{b}$

Ex $a = 3$, $b = 6$, $a < b$
$3 < 6$
both positive

$3 < 6 \Rightarrow \frac{1}{3} > \frac{1}{6}$

Ex $a = -6$, $b = -3$, $-6 < -3$
both negative

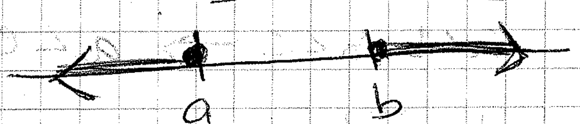
$\frac{1}{-6} > \frac{1}{-3}$

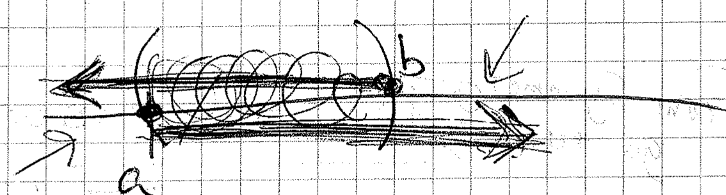


"Sets" $\equiv$ intervals

Review $\cup$, $\cap$
union of sets, intersection of sets

Imagine a solution exists:





$x \leq a$ or $x \geq b$

$(-\infty, a] \cup [b, \infty)$

unconnected sets
"disjoint"

I "An element x is either in set A or it's in set B"

$x \leq b$ and $x \geq a$

$(-\infty, b] \cap [a, \infty)$

intersection
$[a, b]$

"An element x is in both set A + B"