

Sec 3.6 HW

$$1. a) \int_{-1}^{\infty} e^{-5x} dx = \lim_{b \rightarrow \infty} \int_{-1}^b e^{-5x} dx = \lim_{b \rightarrow \infty} \left. -\frac{e^{-5x}}{5} \right|_{-1}^b$$

$$= \lim_{b \rightarrow \infty} \frac{-e^{-5(b)}}{5} - \frac{-e^{-5(-1)}}{5}$$

$$= \lim_{b \rightarrow \infty} \frac{-e^{-5b}}{5} + \frac{e^5}{5} \rightarrow \text{const}$$

$$= \lim_{b \rightarrow \infty} \frac{-1}{e^{5b}} + \text{const} = \frac{-1}{e^{\infty}} + \text{const}$$

$$= 0 + \text{const} = \frac{e^5}{5}$$

Converges to $e^5/5$

$$c) \int_{-\infty}^0 e^{\frac{x}{2}} dx = \lim_{b \rightarrow -\infty} \int_b^0 e^{\frac{x}{2}} dx = \lim_{b \rightarrow -\infty} \left. 2e^{\frac{x}{2}} \right|_b^0$$

$$= 2e^0 - 2e^{\frac{b}{2}} = 2 - 2e^{\frac{b}{2}}$$

$$= 2 \cdot 1 - \frac{2}{e^{\infty/2}} = 2 - 0 = 2$$

Converges to 2

e)
$$\int_{-\infty}^{\infty} \frac{x}{x^2+2} dx = \int_{-\infty}^0 \frac{x}{x^2+2} dx + \int_0^{\infty} \frac{x}{x^2+2} dx$$

$u = x^2 + 2, \quad du = 2x dx$

$u = 0^2 + 2 \quad du = x dx$

$u = 2$

u-sub is: $\int u^n du$ split up the $(-\infty, \infty)$

$$\int_{-\infty}^0 \frac{du}{2u} + \int_0^{\infty} \frac{du}{2u}$$

$$= \lim_{a \rightarrow -\infty} \left. \frac{1}{2} \ln |u| \right|_a^2 + \lim_{b \rightarrow \infty} \left. \frac{1}{2} \ln |u| \right|_2^b$$

$$= \lim_{a \rightarrow -\infty} \left(\frac{\ln 2}{2} - \frac{\ln |-a|}{2} \right) + \lim_{b \rightarrow \infty} \left(\frac{\ln b}{2} - \frac{\ln 2}{2} \right)$$

$\parallel \quad \parallel$
 $\infty \quad \infty$

Note: You don't have to split this at zero. Any const is fine.

Once you have one \int diverging, the entire sum diverges.

Rule $\sqrt{ab} = \sqrt{a}\sqrt{b}$

2. c) $\int_1^{\infty} \frac{dx}{\sqrt{3x}}$ If $u = 3x$, $du = 3dx$

Rewrite $\frac{1}{\sqrt{3}} \int_1^{\infty} \frac{dx}{\sqrt{x}} = \frac{1}{\sqrt{3}} \cdot 2\sqrt{x} \Big|_1^{\infty}$

$\lim_{b \rightarrow \infty} \frac{2}{\sqrt{3}} \sqrt{x} \Big|_1^b = \frac{2}{\sqrt{3}} \cdot \sqrt{\infty} - \frac{2}{\sqrt{3}} = \infty$
diverges

d) $\int_{-\infty}^0 \frac{dx}{(x-2)^3}$ Probably converges

$y = \lim_{x \rightarrow \infty} \frac{1}{(x-2)^3} = \frac{1}{\infty} = 0$

$\lim_{b \rightarrow -\infty} \int_b^0 \frac{dx}{(x-2)^3} = \lim_{b \rightarrow -\infty} \int_{b-2}^{-2} u^{-3} du$

Let $u = x-2$, $u = 0-2 = -2$
 $du = dx$

$= \lim_{b \rightarrow -\infty} \left(\frac{u^{-2}}{-2} \right) \Big|_b^{-2}$

$= \lim_{b \rightarrow -\infty} \left(\frac{-1}{2(-2)^2} - \frac{-1}{2b^2} \right)$

$= \frac{-1}{8} - 0 = -\frac{1}{8}$

11.11.2020

1. $\frac{1}{x^2} = x^{-2}$

2. $\frac{1}{x^3} = x^{-3}$

3. $\frac{1}{x^4} = x^{-4}$

4. $\frac{1}{x^5} = x^{-5}$

5. $\frac{1}{x^6} = x^{-6}$

6. $\frac{1}{x^7} = x^{-7}$

7. $\frac{1}{x^8} = x^{-8}$

8. $\frac{1}{x^9} = x^{-9}$

9. $\frac{1}{x^{10}} = x^{-10}$

10. $\frac{1}{x^{11}} = x^{-11}$

11. $\frac{1}{x^{12}} = x^{-12}$

12. $\frac{1}{x^{13}} = x^{-13}$

13. $\frac{1}{x^{14}} = x^{-14}$

14. $\frac{1}{x^{15}} = x^{-15}$

15. $\frac{1}{x^{16}} = x^{-16}$

16. $\frac{1}{x^{17}} = x^{-17}$

17. $\frac{1}{x^{18}} = x^{-18}$

18. $\frac{1}{x^{19}} = x^{-19}$

19. $\frac{1}{x^{20}} = x^{-20}$

20. $\frac{1}{x^{21}} = x^{-21}$

21. $\frac{1}{x^{22}} = x^{-22}$

g) $\int_2^{\infty} \frac{dx}{x \ln x}$ Let $u = \ln x$ $n = -1$
 $du = \frac{dx}{x}$

$$\int u^{-1} du = \ln|u| + C$$

$$= \ln|\ln|x|| + C$$

$\lim_{b \rightarrow \infty} \ln|\ln|2| - \ln|\ln|b||$
slowly to ∞

ring # \rightarrow slowly to $\infty = -\infty$
 \therefore divergent

b) $\int_1^{\infty} \frac{dx}{x^3} = ?$

$\int_1^{\infty} \frac{dx}{x^2} = 1$

~~$\int \frac{1}{x^1} / \int \frac{1}{\sqrt{x}}$~~

$\lim_{b \rightarrow \infty} \left. \frac{-x^{-2}}{2} \right|_1^b = \left. \frac{-1}{2x^2} \right|_1^b$

$\lim_{b \rightarrow \infty} \left(\frac{1}{2} - \frac{1}{2 \cdot b^2} \right)$
 $\rightarrow \frac{1}{2} + \frac{1}{2} = \frac{1}{2}$

1-20

$$x \cdot \frac{1}{x} = 1$$

$$\frac{1}{x} = x^{-1}$$

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$$\frac{1}{x^2} = x^{-2}$$

$$\frac{1}{x^3} = x^{-3}$$

$$\frac{1}{x^4} = x^{-4}$$

$$\frac{1}{x^5} = x^{-5}$$

...

$$\frac{1}{x^6} = x^{-6}$$

$$\frac{1}{x^7} = x^{-7}$$

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$$\frac{1}{x^8} = x^{-8}$$

$$\frac{1}{x^9} = x^{-9}$$

$$\frac{1}{x^{10}} = x^{-10}$$

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