Sec 36 HW

1. a) $\int_{-1}^{\infty} e^{-5x}\,dx = \lim_{b\to\infty} \int_{-1}^{b} e^{-5x}\,dx = \lim_{b\to\infty} -\frac{e^{-5x}}{5}\Big|_{-1}^{b}$

$= \lim_{b\to\infty} \frac{-e^{-5(b)}}{5} - \frac{-e^{-5(-1)}}{5}$

$= \lim_{b\to\infty} -\frac{e^{-5b}}{5} + \frac{e^{5}}{5} \longrightarrow$ const

$= \lim_{b\to\infty} -\frac{1}{e^{5b}} + \text{const} = \frac{-1}{e^{\infty}} + \text{cost}$

$= 0 + \text{const} = \boxed{\frac{e^5}{5}}$ Converges to $e^5/5$

c) $\int_{-\infty}^{0} e^{\frac{x}{2}}\,dx = \lim_{b\to-\infty} \int_{b}^{0} e^{\frac{x}{2}}\,dx = \lim_{b\to-\infty} 2e^{\frac{x}{2}}\Big|_{b}^{0}$

$= 2e^{0} - 2e^{-\infty/2}$ $-\infty/2 = -\infty$

$= 2\cdot 1 - \frac{2}{e^{\infty/2}} = 2 - 0 = 2$

Converges to 2

e) $$\int_{-\infty}^{\infty} \frac{x}{x^2+2}\,dx = \int_{-\infty}^{0} \frac{x}{x^2+2}\,dx + \int_{0}^{\infty} \frac{x}{x^2+2}\,dx$$

$u = x^2 + 2$, $du = 2x\,dx$

$u = 0^2 + 2$ $\quad \frac{du}{2} = x\,dx$

$u = 2$

split up the $(-\infty, \infty)$

u-sub is: $\int u^n\,du$

$$\boxed{\int_{-\infty}^{0} \frac{du}{2u} + \int_{0}^{\infty} \frac{du}{2u}}$$

$$= \lim_{a \to \infty} \frac{1}{2} \ln|u| \Big|_{a}^{2} + \lim_{b \to \infty} \frac{1}{2} \ln|u| \Big|_{2}^{b}$$

$$= \lim_{a \to -\infty} \cancel{\frac{\ln 2}{2}} - \underbrace{\frac{\ln|-\infty|}{2}}_{\infty} + \lim_{b \to \infty} \underbrace{\frac{\ln \infty}{2}}_{\infty} - \cancel{\frac{\ln 2}{2}}$$

Note: You don't have to split this at zero. Any const is fine.

Once you have one $\int$ diverging, the entire sum diverges.

Rule $\sqrt{ab} = \sqrt{a}\sqrt{b}$

2, c) $\displaystyle\int_1^\infty \frac{dx}{\sqrt{3x}}$ If $u = 3x$, $du = 3dx$

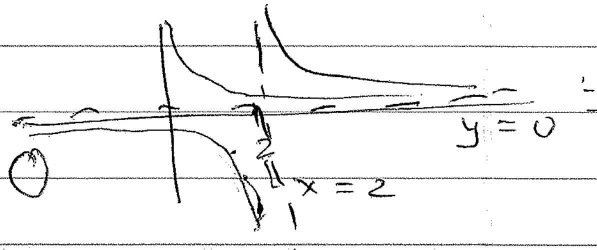
Rewrite $\displaystyle\frac{1}{\sqrt{3}} \int_1^\infty \frac{dx}{\sqrt{x}} = \frac{1}{\sqrt{3}} \cdot 2\sqrt{x} \Big|_1^\infty$

$\displaystyle\lim_{b \to \infty} \frac{2}{\sqrt{3}} \sqrt{x} \Big|_1^b = \frac{2}{\sqrt{3}} \cdot \sqrt{\infty} - \frac{2}{\sqrt{3}} = \infty$

diverges

d) $\displaystyle\int_{-\infty}^0 \frac{dx}{(x-2)^3}$ Probaby converges

$\displaystyle y = \lim_{x \to \infty} \frac{1}{(x-2)^3} = \frac{1}{\infty} = 0$



$\displaystyle\lim_{b \to -\infty} \int_b^0 \frac{dx}{(x-2)^3} = \lim_{b \to -\infty} \int_{b-2}^{-2} u^{-3}\,du$

Let $u = x - 2$, $u = 0 - 2 = -2$

$du = dx$

$\displaystyle = \lim_{b \to -\infty} \left(\frac{u^{-2}}{-2} \right) \Big|_b^{-2}$

$\displaystyle = \lim_{b \to -\infty} \left(\frac{-1}{2(-2)^2} - \frac{-1}{2b^2} \right)$ $\to 0$ $+\infty$

$\displaystyle = \frac{-1}{8} - 0 = -\frac{1}{8}$

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g) $\displaystyle\int_{2}^{\infty} \frac{dx}{x\ln x}$ Let $\boxed{u = \ln x}$ $n=-1$

$du = \dfrac{dx}{x}$

$$\boxed{\begin{aligned}\int u^{-1}\,du &= \ln|u| + C \\ &= \ln\left|\ln|x|\right| + C\end{aligned}}$$

$$\lim_{b\to\infty} \ln|\ln|2|| - \underbrace{\ln|\underbrace{\ln \infty}_{\text{slow to } \infty}|}$$

Tiny # $-$ slowly to ∞ $= -\infty$

$\therefore$ divergent

b) $\displaystyle\int_{1}^{\infty} \frac{dx}{x^3} = ?$ $\displaystyle\int_{1}^{\infty} \frac{dx}{x^2} = 1$

$\cancel{\int \frac{1}{x^1}, \int \frac{1}{\sqrt{x}}}$

$$\lim_{b\to\infty} -\frac{x^{-2}}{2}\Bigg|_1^b = \lim_{b\to\infty} \frac{-1}{2x^2}\Bigg|_1^b$$

$$\lim_{b\to\infty} -\frac{1}{2b^2} - \frac{-1}{2\cdot 1^2}$$

0

$0 + \dfrac{1}{2} = \dfrac{1}{2}$