

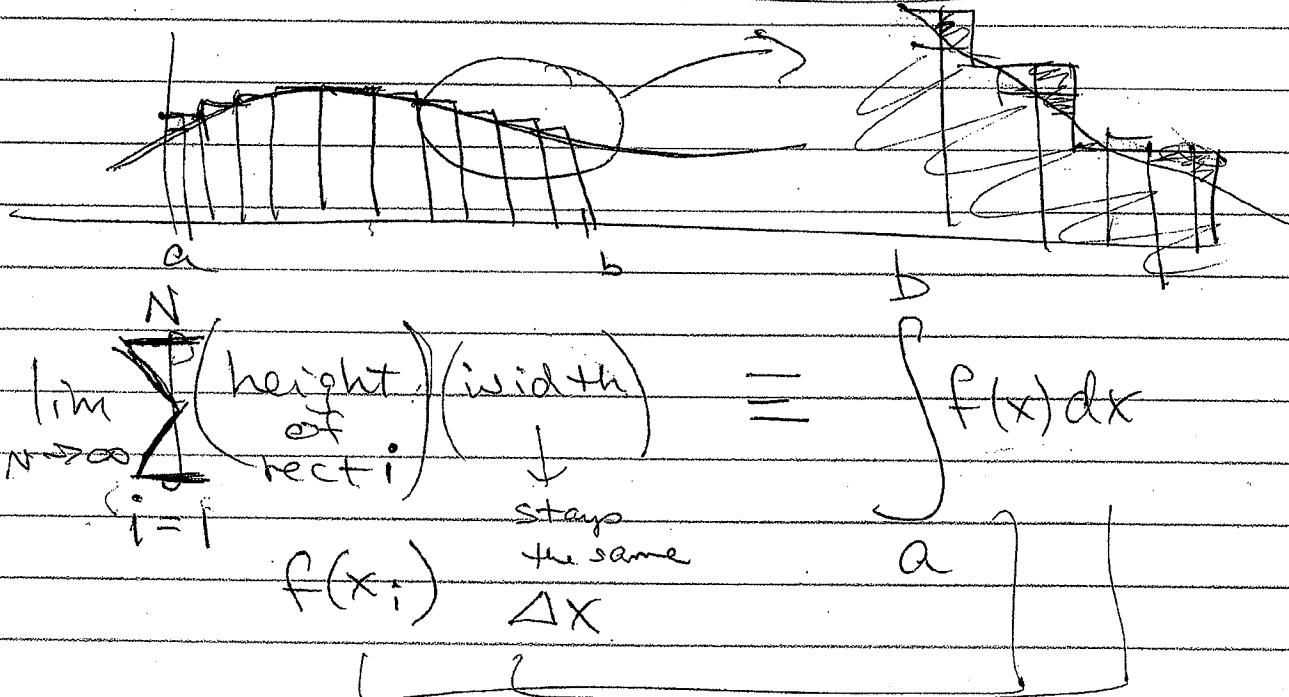
~~POST~~

(A)

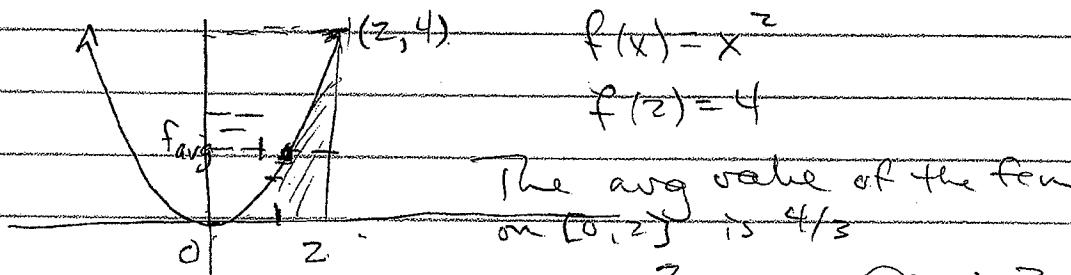
Sec. 35 is 3 sections

I Riemann sum
on $[a, b]$

$$= \lim_{N \rightarrow \infty} \sum_{i=1}^N f(x_i) \Delta x = \int f(x) dx$$



Ex 35.1 Avg value of $f(x) = x^2$ on $[0, 2]$



$$\text{Area} = \int_0^2 x^2 dx = \left[\frac{x^3}{3} \right]_0^2 = \frac{8}{3}$$

$$\text{Avg value} = \frac{1}{b-a} (\text{area}) = \frac{1}{2-0} \cdot \frac{8}{3} = \frac{8}{6} = \frac{4}{3}$$

Sec 35 #1 continued

#1d) $f(x) = \sqrt{x+1}$ on $[1, 2]$

~~$f(x) = \frac{1}{2} \int_{-1}^x (\sqrt{x+1}) dx = 2(x+1)^{3/2} \Big|_1^2$~~

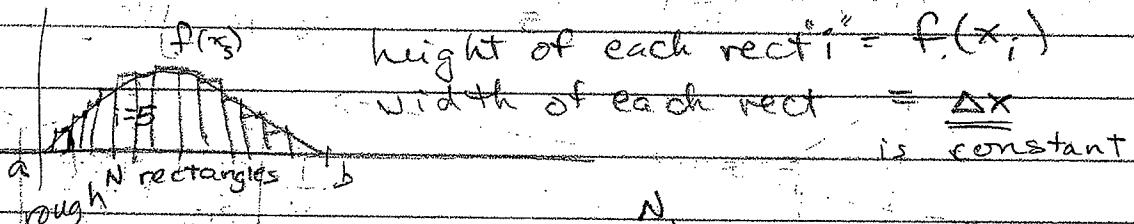
~~on $[1, 2]$~~ $= \frac{2}{3} (3^{3/2} - 1^{3/2}) = \frac{2}{3} (3^{3/2} - 1)$ This is

Post

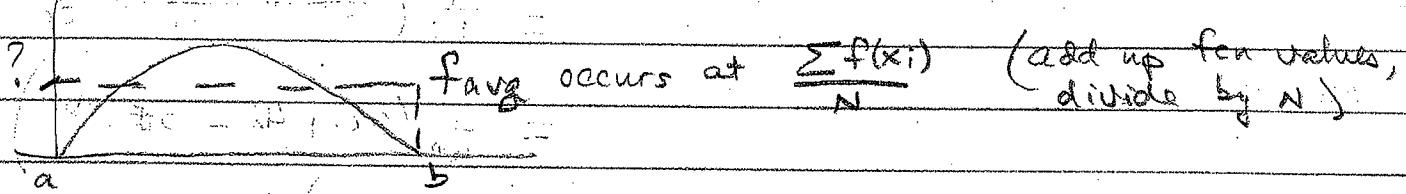
(B)

Sec 35 - Several formulas + applications

II. Average value of a function on $[a, b]$



The sum of func. values is $\sum_{i=1}^N f(x_i)$, where $f(x_i)$ is the func. value at the top of the i th rectangle.



$$\text{Notice that } \frac{\sum f(x_i) \cdot \Delta x}{N \cdot \Delta x} = \frac{\sum f(x_i)}{N}$$

and that $b-a = N\Delta x$, the sum of lengths of rect bases
Hence, the average value of the func on $[a, b]$ is roughly

$$\frac{1}{N\Delta x} \sum_{i=1}^N f(x_i) \Delta x = \frac{1}{b-a} \sum_{i=1}^N f(x_i) \Delta x \quad \text{Rough est of avg value}$$

The exact average value of f on $[a, b]$ is found by taking the limit of these rect. areas (i.e., the integral $\int_a^b f(x) dx$) and multiplying by $\frac{1}{b-a}$

$$\lim_{N \rightarrow \infty} \frac{1}{b-a} \sum_{i=1}^N f(x_i) \Delta x = \cancel{\int_a^b f(x) dx}$$

$$\text{Avg value of } f(x) \text{ on } [a, b] = \frac{1}{b-a} \int_a^b f(x) dx \quad \star$$

HW See 35

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$$\#3. \quad H(t) = \frac{1}{10}t + \frac{12}{5}$$

$$\frac{1}{10} \int_{0}^{20} \left(\frac{1}{10}t + \frac{12}{5} \right) dt$$

$H(t)$
(millions)

1960 1970 1980
 $t=0$ $t=10$ $t=20$

$$\frac{1}{10} \left[\frac{t^2}{20} + \frac{12t}{5} \right] \Big|_0^{20}$$

$$= \frac{1}{10} \left(\frac{20^2}{20} + \frac{12(20)}{5} - \frac{10^2}{20} - \frac{12(10)}{5} \right)$$

Avg. value
of $f(x)$
on $[a, b]$

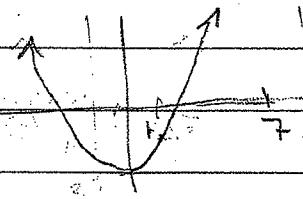
$$= \frac{1}{b-a} \int_a^b f(x) dx$$

$$= \frac{1}{10} (20 + 48 - 50) = 24$$

$$= \frac{1}{10} (39) = 3.9 \text{ million hamburgers}$$

H2

$$\#1b) \quad f(x) = x^2 - 3 \quad \text{Find } f_{\text{avg}} \text{ on } [1, 7]$$



Notice the interval straddles a root
($x^2 - 3 = 0 \rightarrow x = \pm \sqrt{3}$)

Does this require we split up the integral?

No, since we are not considering area.

Hence, $f_{\text{avg}} = \frac{1}{7-1} \int_1^7 (x^2 - 3) dx = \frac{1}{6} \left[\frac{x^3}{3} - 3x \right] \Big|_1^7$ notice the use of brackets

$$= \frac{1}{6} \left[\left(\frac{7^3}{3} - 3 \cdot 7 \right) - \left(\frac{1^3}{3} - 3 \cdot 1 \right) \right] = \frac{1}{6} \left(\frac{343}{3} - 63 + \frac{8}{3} \right)$$

$$= \frac{1}{6} \left(\frac{343 - 189 + 8}{3} \right) = \frac{96}{6} = \boxed{16}$$

(C)

See 35 - continued (topic III)

III

Present + Accumulated Value - Previously, we considered an investment of a principal at some annual rate of interest, compounded continuously, and we sought its value at some future time.

$$FV = Pe^{rt}$$

This scenario assumes a one-time input of money, with growth on interest and principal alone. More realistically, money is flowed into an investment throughout the years of investment.

Without continuous flow, the above formula solved for P would tell us the "present value" of an investment that at some future time t is worth FV.

$$P = FV e^{-rt}$$

That is, to attain a given FV, at some rate of time of investment, you'd need $P = FV e^{-rt}$ principal. This "P" we now call "present value".

$$PV = FV e^{-rt}$$

If money is flowing continuously at some $f(t)$, i.e.
the analogous formulas are these:

$$PV = \int_0^T f(t) e^{-rt} dt \quad FV = \int_0^T f(t) e^{r(T-t)} dt$$

↓ How it generates income
Calculate this when you need to compare your business investment to an offer to buy, i.e., fair market price.

↓ income flowing often a constant stream
Generally the scenario for bank investment (deposits)

∴ PV is more common for deciding to sell or keep a business

FV is the choice for looking at value of an investment.