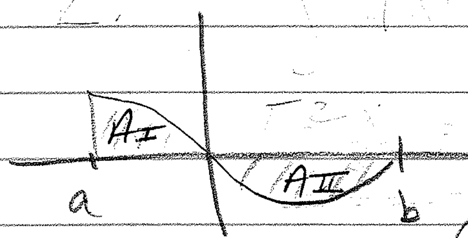
Sec 34 - Definite Integral ("signed area") vs Area



$A_I + A_{II}$

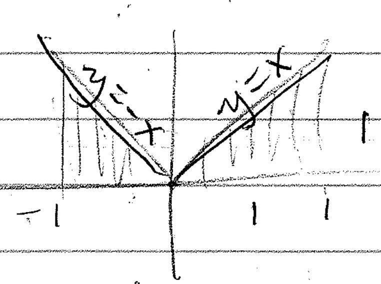
① $\int_a^b f(x)\,dx$ = net area (Area above − Area below)
"signed" − THE DEF INT

② $\int_a^0 f(x)\,dx + \left(- \int_0^b f(x)\,dx\right)$ = actual area

We could write the second as $\int_a^0 f(x)\,dx + \int_b^0 f(x)\,dx$

The actual area = abs value of the integrals

$f(x) = |x|$



$\text{Area} = \int_{-1}^{1} |x|\,dx = \int_{-1}^{0} -x\,dx + \int_{0}^{1} x\,dx$

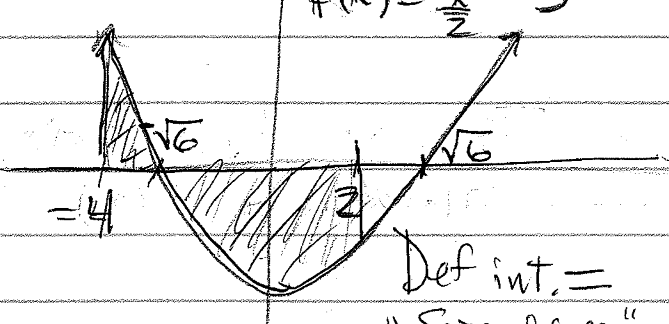
$= 1$

Here the area is the same as the integral, but b/c it's piecewise fcn, we have to break it up.

piecewise

"Area of f(x) on [−4, 2]

$f(x) = \frac{x^2}{2} - 3$



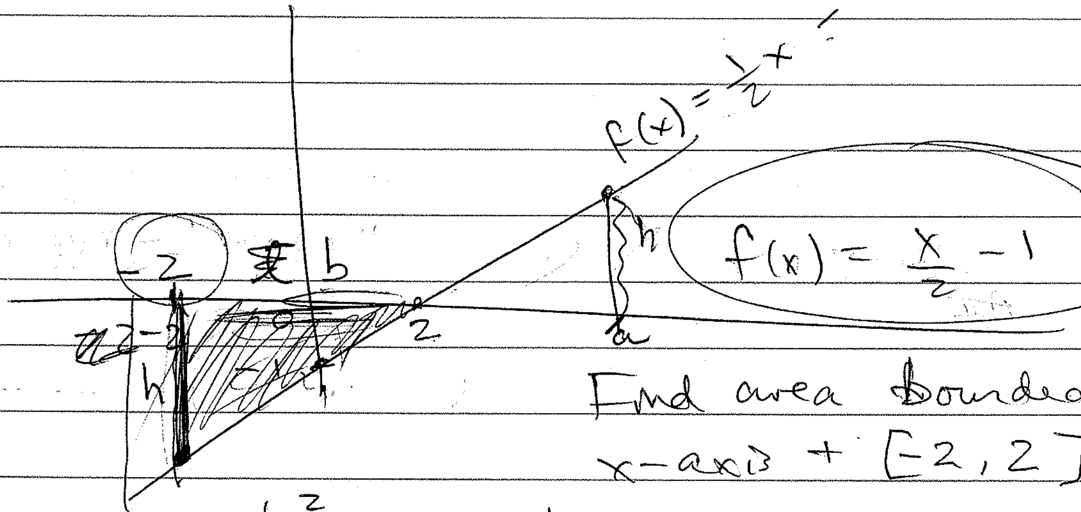
$\int_{-4}^{2} \left(\frac{x^2}{2} - 3\right) dx$ = "net area" "signed area" (def. integral)

Def int. = $\int_{-4}^{-\sqrt{6}} \left(\frac{x^2}{2} - 3\right) dx + \underbrace{\int_{-\sqrt{6}}^{2} \left(\frac{x^2}{2} - 3\right) dx}_{\text{negative b/c below x-axis}}$

"Signed area"
Net area

$\boxed{\text{Actual Area}} = \int_{-4}^{-\sqrt{6}} f(x)\,dx - \int_{-\sqrt{6}}^{2} f(x)\,dx$ neg

make the integral (+)



$f(x) = \frac{x}{2} - 1$

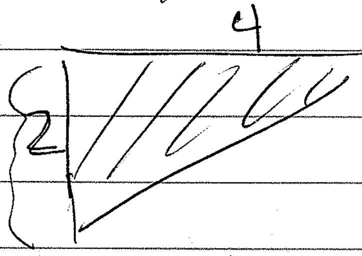
Find area bounded by x-axis + $[-2, 2]$

$$\text{Area} = \left| \int_{-2}^{2} f(x)\,dx \right| = \int_{-2}^{2} \left(\frac{x}{2} - 1\right) dx$$

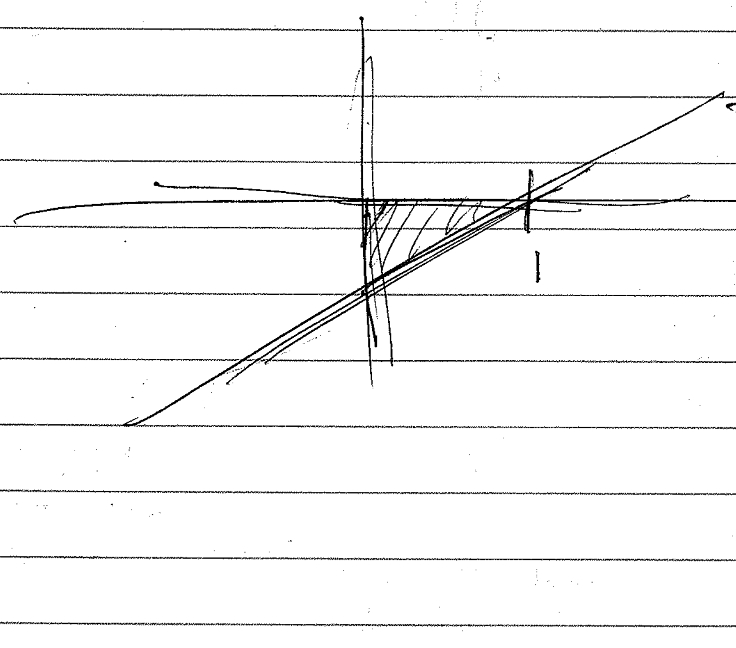
$$= \left| \frac{x^2}{4} - x \,\Big|_{-2}^{2} \right| = \left| \left(\frac{4}{4} - 2\right) - \left(\frac{4}{4} - -2\right) \right|$$

$$= \left| \cancel{1} - 2 - \cancel{1} - 2 \right| = |-4| = 4$$

$f(-2)$



$$|f(-2)| = \frac{1}{2}(-2) - 1$$
$$= -1 - 1 = |-2|$$
$$= 2$$



Bounded by x-axis, y-axis + f(x)