Sec 33 - Definite Integrals.

Recall that $\int f(x)\,dx = F(x) + C$

where the derivative of the fcn on the left is $f(x)$

That is, $\frac{d}{dx}\int f(x)\,dx = \frac{d}{dx}\left(F(x)+C\right) = F'(x) + 0$

Hence, $F(x) + C$ is the family of antiderivatives of $f(x)$.
~~Taking~~ Finding an antiderivative of $f(x)$ is also called finding an indefinite integral of a fcn. $f(x)$

Before, we've considered $f(x)$ to be a continuous fcn. on an ~~open~~ interval, generally, its natural domain. So

$\int (x^2+2x+1)\,dx$ is the family of antiderivatives

$\frac{x^3}{3} + x^2 + x + C$ on $\mathbb{R}$.

Since ultimately we want a definite answer to $\int f(x)\,dx$ on some closed interval $[a,b]$ for application purposes, we now consider the algorithm that effects (brings about) this answer.

Def The definite integral of a cts. fcn. $f(x)$ on $[a,b]$ is given by

$$\int_a^b f(x)\,dx \overset{def}{\equiv} F(b) - F(a),$$

a constant, not a fcn

Notice the "C" isn't needed. It's in the def. as

$\int f(x)dx$ eval at $x = a \Rightarrow F(a) + C$

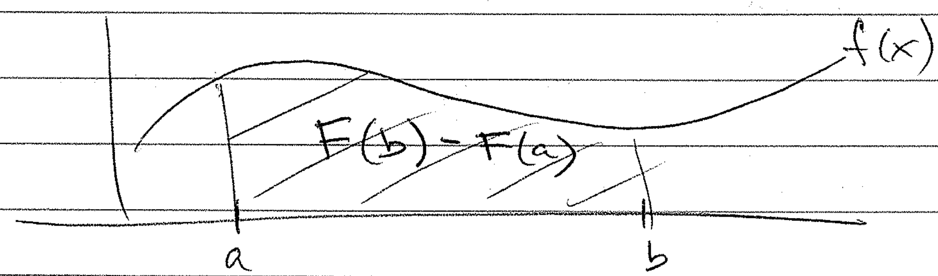
$\int f(x)dx$ " " $x = b \Rightarrow F(b) + C$

so $\left(F(b)+C\right) - \left(F(a)+C\right) = F(b) - F(a)$

No need to write $C - C = 0$.

In $\int_a^b f(x)dx$, a & b are the limits of integration.

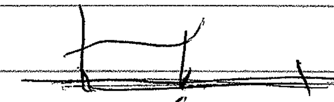
They have to be in the domain of f(x) or the integral makes no sense, since physically, integration is this



Although we won't investigate this fully till Sec 34 (Mon)

Rules of def. integrals

- $\int_a^a f(x)dx = 0 \quad (F(a) - F(a) = 0)$
- $k\int_a^b f(x)dx = k(F(b) - F(a)) = \int k f(x)dx = k\int f(x)dx$
- $\int_a^b f(x) \pm g(x)\,dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx$



- For any c s.t. $a \le c \le b$, $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$

$$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$$

since $F(b) - F(a) = -\left(F(a) - F(b)\right)$

How do we deal with the limits of integration when we have the processes of u-sub? It's a very easy tactic:

ex $\displaystyle\int_1^2 \frac{x^2}{x^3+1}\,dx$ This looks like a u-sub situation (see the degrees of top + bottom)

Let $\boxed{u = x^3+1}$ so $\dfrac{du}{dx} = 3x^2$, ~~or du = 3x dx~~

or $du = 3x^2\,dx$

and $x^2\,dx = \dfrac{du}{3}$

Sustituting:

$$\int_{a=(x=)1}^{b=(x=)2} \frac{x^2}{x^3+1}\,dx = \int_{?}^{?} \frac{1}{u}\cdot\frac{du}{3}$$

Write the limits in terms of u

What are the limits of integration in terms of u?

Simply use the $\boxed{u = x^3+1}$ and substitute each x:

$a = (u =) 1^3 + 1 = 2$, $b = (u =) 2^3 + 1 = 9$

$$\int_2^9 \frac{du}{3u} = \frac{1}{3}\int_2^9 \frac{du}{u} = \frac{1}{3}\left(\ln u\right)\Big|_2^9$$

$$= \frac{1}{3}\ln 9 - \frac{1}{3}\ln 2 = \boxed{\frac{1}{3}\ln\frac{9}{2}}$$