0

$$\int f(x)\,dx \longrightarrow k\int u^n\,du = k\frac{u^{n+1}}{n+1} + C$$

$$\int e^{f(x)}\,dx \longrightarrow k\int e^u\,du = ke^u + C$$

After you establish these u forms, integrate the obvious way, then resubstitute the original $f(x)$ in the answer.

Sec 31 HW #1a-n, #3, #4, #5

Sec 32

$$\int x\,dx = \frac{1}{5}\int 5x\,dx$$

use this notation when we force $\int f(x)\,dx$ into $k\int u^n\,du$

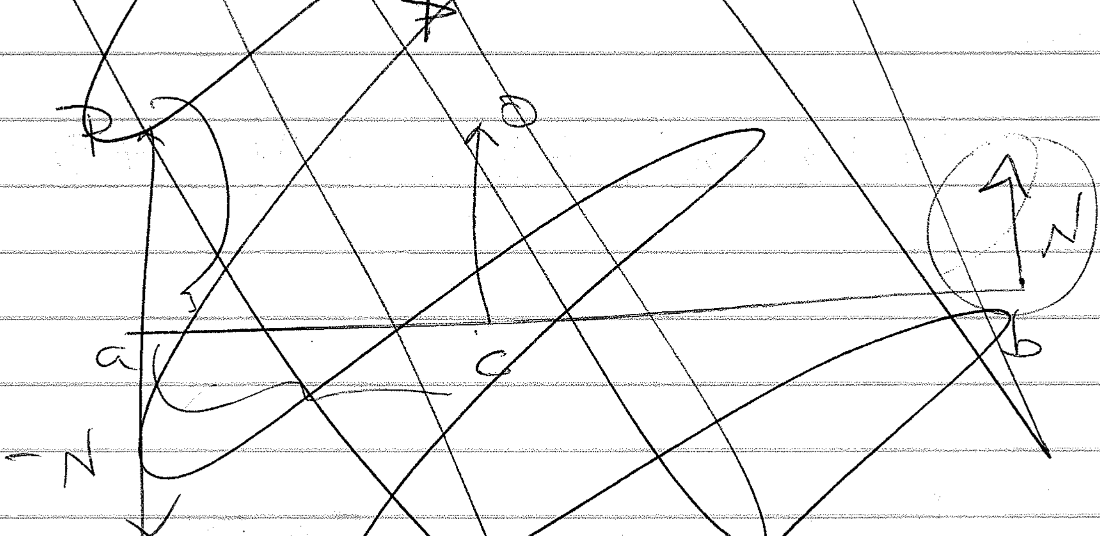
$Pac = Nac + Nbc$

$(P-N)ac = Nbc$

$Pac - Nac = Nbc$

$Pac = Nac + Nbc$

$ac = \dfrac{N(ac + bc)}{P}$



Moment of N-N = $\cancel{Pac}$ $N(ac + bc)$

Sec 31

Revisited

u-substitution Ex. 31:4

$$\underbrace{\int e^{\overbrace{x^3-3x}^{u}}}_{\text{factor 1}} \underbrace{(x^2-1)\,dx}_{\text{factor 2}} \longrightarrow \int e^u \underbrace{(x^2-1)\,dx}_{\text{Is this } du?}$$

If the 2 factor terms in the integrand differs from each other by a constant factor, we can adjust the integrand by that factor, as follows: to get $\int u^n\,du$

$$\int \underbrace{e^{x^3-3x}}_{e^u} \cdot \underbrace{x^2-1 \cdot dx}_{du = ?}$$

where $u = x^3-3x$

and $\frac{du}{dx} = 3x^2-3 = 3(x^2-1)$

multiply sides by dx:

$$du = \underbrace{3}\,\underbrace{(x^2-1)\,dx}$$

This is equal to the current factor $(x^2-1)dx$ except for the multiplier of 3. So we adjust the integrand like this (a little different algorithm from your book):

$$\int e^u (x^2-1)\,dx = \frac{1}{3}\int e^u \underbrace{3(x^2-1)\,dx}_{du}$$

$$= \frac{1}{3}\int e^u\,du$$

~~Itu~~ Integrating the u-substituted form:

$$\frac{1}{3}\int e^u\,du = \frac{1}{3}\left(e^u + C\right) = \boxed{\frac{1}{3}e^{x^3-3x} + C}$$

NOTE - An integrand can only be adjusted by a constant factor, not by ⊕, ⊖, squaring, of a variable or constant. You may only ⊗, ⊘ by a constant.

Ex 31.5 $\int \frac{6x+12}{x^2+4x}\,dx \longrightarrow k\int u^n\,du$

First, factor out any obvious const. multiple

$6\int \frac{x+2}{x^2+4x}\,dx$ Which of these is the obvious u so the other is $\frac{du}{dx}$?

Let $u = x^2+4x$; then

$\frac{du}{dx} = 2x+4 \longrightarrow \boxed{du = (2x+4)\,dx = 2(x+2)\,dx}$

In $6\int \frac{x+2}{x^2+4x}\,dx$, the top $(x+2)\,dx$

is off only by a factor of 2. Adjusting:

$$6\cdot\frac{1}{2}\int\frac{2(x+2)\,dx}{x^2+4x} = \boxed{3\int\frac{du}{u}} = 3\ln|u| + C$$

$$= 3\ln|u| + C = 3\ln|x^2+4x| + C$$

Differentiate to check that you get the original integrand.

Remember, $3\ln|x^2+4x|+C$ is the antiderivative or indefinite integral of $g(x) = \dfrac{6x+12}{x^2+4x}$

Look at Ex 31.8. You'll be tempted to try a u-substitution, but it won't work. However, simply expanding and breaking up the polynomial does the trick.

$$\int \underbrace{x^2}_{u} (2x+3)\,dx$$

u, $\frac{du}{dx} = 2x$, not $2x + 3$ You can't adjust by a const.

$$\int 2x^3\,dx + \int 3x^2\,dx = \frac{\cancel{2}x^4}{\cancel{4}\,2} + \frac{\cancel{3}x^3}{\cancel{3}} + C$$

Ex. 31.6 + 7 Sometimes you have to do an intermediate step to see the u-substitution

Ex 31.6 $\int x^3(x^2+1)^{3/2}\,dx$ You generally choose the term with the higher power of x to be the u-term.

Ex 31.6 $\int x^3 (x^2+1)^{3/2}\,dx$ This requires some trial + error

(4)

letting $u = x^3$	letting $u = x^2+1$
$u = x^3$	$u = x^2+1$
$\frac{du}{dx} = 3x^2$	$\frac{du}{dx} = 2x$
$du = 3x^2\,dx$ which is not off by a const. multiplier from x^2+1 (let alone $(x^2+1)^{3/2}$)	$du = 2x\,dx$
Forget this.	But since $u = x^2+1$, $u - 1 = x^2$, $du = 2x\,dx$. Look at the integrand, replacing what you can:

$$\int x^3 (x^2+1)^{3/2}\,dx$$

almost $2x\,dx$

$$= \int \underbrace{x^2}_{u-1} \cdot x \cdot \underbrace{(x^2+1)^{3/2}}_{u}\,dx$$

Multiply by 2 inside, by $\frac{1}{2}$ outside:

$$\frac{1}{2}\int \underbrace{x^2}_{u-1} \cdot \boxed{2x}\, \underbrace{(x^2+1)}_{u}{}^{3/2}\, \boxed{dx}$$

du

$$= \frac{1}{2}\int (u-1)\, u^{3/2}\,du = \frac{1}{2}\int \left(u^{5/2} - u^{3/2}\right) du$$

$$= \frac{1}{2}\left(\frac{u^{7/2}}{7/2} - \frac{u^{5/2}}{5/2}\right) + C$$

$$= \frac{u^{7/2}}{7} - \frac{u^{5/2}}{5} + C$$

$$= \frac{(x^2+1)^{7/2}}{7} - \frac{(x^2+1)^{5/2}}{5} + C$$

Ex 31.7 $\displaystyle\int \frac{x}{x+1}\,dx$

Choose u + then solve for x

Simply: Let $\boxed{u = x+1}$, then $\boxed{x = u-1}$ and $\boxed{du = dx}$

Substitute $\displaystyle\int \frac{u-1}{u}\,du = \int \frac{u}{u}\,du - \int \frac{du}{u}$

$$= \int du - \int \frac{du}{u} = u - \ln|u| + C$$

$$= x+1 - \ln|x+1| + C$$

Sec 32 Integration by Parts

Rather than seeing $\int f(x)\,dx$ as $\int u\,du$

we're viewing $\int f(x)\,dx$ as $\int u\,dv$

and applying the formula

$$\boxed{\int u \underbrace{dv}_{\text{name}} = uv - \int v\,du}$$ Memorize

where we expect $\int v\,du$ to be an easier integral than $\int u\,dv$

Background — from product rule:

$$\boxed{\frac{d}{dx}(uv)} = u \cdot \frac{dv}{dx} + v\frac{du}{dx}$$

Rewriting: $u\frac{dv}{dx} = \frac{d}{dx}(uv) - v\frac{du}{dx}$

Integrate both sides with respect to x:

Why the #?#! is she doing this?

$$\int u \frac{dv}{\cancel{dx}} \cdot \cancel{dx} = \boxed{\int \frac{d}{\cancel{dx}}(uv)\cancel{dx}} - \int v \frac{du}{\cancel{dx}} \cdot \cancel{dx}$$

$$\int u\,dv = uv - \int v\,du$$

the antiderivative of the derivative is the fcn.

Ex $\int x e^x \, dx$ becomes $\int u \, dv$ where we hope if we let $u = x$ and $\underbrace{dv = e^x dx}$ leads to an integrable $\underbrace{uv - \int v \, du}$

Notice: dv + dx stay together

Setting up all the parts:

$u = x, \quad dv = e^x dx$

$du = dx, \quad v = ?$ This is gotten by integrating both sides

$\int dv = \int e^x dx \longrightarrow \boxed{v = e^x}$ (the "+ C" will be subsumed into the final C)

Finally:

$$\int x e^x dx = x e^x + \int e^x dx$$

$$= \boxed{x e^x + e^x + C}$$

Keys 1. Choose u + dv so $\int v \, du$ is doable.

2. Choose dv so $\int dv$ is known

Ex 32.3 $\qquad \int \ln x \, dx$

We don't know how to do this yet directly (We know $\frac{d}{dx}(\ln x) = \frac{1}{x}$ but not $\int \ln x \, dx$)

Very simply, let $\overbrace{u = \ln x, \quad dv = dx}^{\text{name}}$

so, $\frac{du}{\cancel{dx}} = \frac{1}{x} dx$, $v = x$

Then: $\int \ln x \, dx = uv - \int v \, du$

$= x \ln x - \int \frac{x}{x} dx$

$$\int \ln x \, dx = \boxed{x \ln|x| - x + C}$$ Good one to know

Ex 32.4 $\qquad \int \frac{\ln x}{x^3} dx$

First try: $u = 1/x^3 \qquad dv = \ln x \, dx$

$du = -3x^{-4} dx, \quad v = ?$

Second try: $u = \ln x, \qquad dv = \frac{dx}{x^3}$

$du = \frac{dx}{x} \qquad v = -\frac{x^{-2}}{2}$