Sec 2

1. Find fcn domain (those values x for which $f(x)$ makes sense, that is, which result in real values $y = f(x)$). Alternately, those values where $f(x)$ is defined.

a) $f(x) = x^3 - 3x^2 + 2x + 5$, $\boxed{\text{Dom } f: \ x \in \mathbb{R}}$

For any polynomial $f(x)$; the fcn is defined for all real numbers.

b) $y = \dfrac{2x-4}{2x+5}$, Since $2x+5 \neq 0$, $x \neq -5/2$

Dom: $\{x \in \mathbb{R} \mid x \neq -5/2\}$

c) $f(x) = \dfrac{4x+2}{x^2}$, Since $x^2 \neq 0$, $x \neq 0$.

Dom: $\{x \in \mathbb{R} \mid x \neq 0\}$

d) $g(t) = \sqrt{t^2+4}$. Since $t^2 + 4 \geq 0$, we see any

Dom. $\longrightarrow$ $\boxed{t \in \mathbb{R}}$ suffices. t^2 is always nonnegative, $t^2 + 4$ moreso.

alternate

e) $f(x) = -\sqrt{\dfrac{5x}{x^2+6}}$ $\longrightarrow$ Uh oh. I copied the question wrong. That's okay; it's instructive.

The negative outside the radical is not important. We need the radicand to be positive and the denominator to be nonzero. Since $x^2 + 6$ is always positive, we only need to check that $5x$ be nonnegative. That is, $5x \geq 0$, or $\boxed{x \geq 0}$ is the domain.

Original 1e) $f(x) = -\sqrt{\dfrac{5}{x^2+6}}$, Dom: $x \in \mathbb{R}$

since $5 > 0$ & $x^2 + 6 > 0$ for all $x \in \mathbb{R}$

f) $f(x) = \sqrt{\dfrac{-5}{x^2+6}}$ The radicand is always negative, so no x will work. $\boxed{\text{Dom: } \varnothing}$ (empty set)

↳ Invalid expression.

g) $f(x) = -\sqrt{\dfrac{5}{x+6}}$ $x + 6 > 0$ or $\boxed{x > -6}$ makes the radicand positive

↳ Dom: $x > -6$

Roots: None, since $5/x+6 = 0$ has no soln.

2. Find domain & all roots:

a) $f(x) = \sqrt{2x-7}$ Dom: $2x - 7 \geq 0$ or $\boxed{x \geq 7/2}$

Roots: $\sqrt{2x-7} = 0$, so $2x - 7 = 0$ or $\boxed{x = 7/2}$

b) $f(x) = \sqrt{5-x}$ Dom $5 - x \geq 0$ or $\boxed{x \leq 5}$

Root: $\sqrt{5-x} = 0$, so $5 - x = 0$ or $\boxed{x = 5}$

c) $f(x) = \dfrac{x^2 + x - 2}{x^2 + 7x + 10}$ Dom: From $x^2 + 7x + 10 \neq 0$

so $(x+5)(x+2) \neq 0$

or $\boxed{x \neq -5, -2}$

Roots: $x^2 + x - 2 = 0$, or $(x-1)(x+2) = 0$; $x = 1$, -2 omit

Discard $x = -2$ as it is not in the domain

But, $x=-2$ is not in the domain, so there's only one root, $\boxed{x=1}$

d) $f(x) = \dfrac{x^2+2}{2x+1}$ Dom: $2x+1 \neq 0$, $\boxed{x \neq -1/2}$

Root: $x^2+2=0$. But x^2+0 is always positive, so there's no root.

e) $f(x) = \dfrac{x^2+3x}{x}$ $\boxed{\text{Dom: } x \neq 0}$

Root: $x^2+3x=0$

$x(x+3)=0$, $x=0$ or -3

But $x \neq 0$, see domain, so $\boxed{x=-3}$ is the root

f) $f(x) = \sqrt[3]{\dfrac{x-2}{x+6}}$ An odd root may be taken of either a positive or negative number (ex: $\sqrt[3]{-8}=-2$)

So almost any $x \in \mathbb{R}$ will do, except $x \neq -6$ (denom $\neq 0$)

$\boxed{\text{Dom: } x \neq -6}$

Root: $\sqrt[3]{\dfrac{x-2}{x+6}} = 0 \longrightarrow \sqrt[3]{x-2} = 0$, $\boxed{x=2}$

g) $f(x) = \sqrt{\dfrac{x}{x+1}}$ An even root (here, a square root) may be taken of a nonnegative number only.

To do this, we check the intervals around the zeros of the numerator & denominator

#2 Domain (alternate problem)

Find x so that

$f(x) = \sqrt{x^2+2x-15}$ D_f: $x^2+2x-15 \geq 0$

<u>Soln</u> You <u>can't</u> find the soln set from the inequality $x^2+2x-15 \geq 0$ directly.

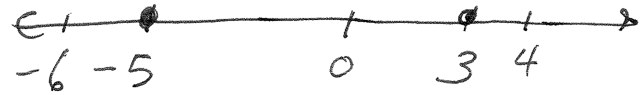
You have to first ~~fill~~ <u>factor</u> the polynomial and then note the zeros of the expression.

$$x^2+2x-15 = (x-3)(x+5) = 0$$

Clearly, $x=3$ and $x=-5$ are the zeros, so these are in the soln set.

Plot these on a number line and inspect values within each of the three intervals, ~~see if the inequality holds~~ where to see where the inequality holds.

Test $x = 0, 4, -6$



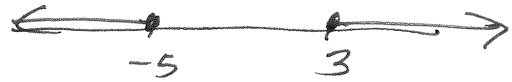
We want those x where $(x-3)(x+5)$ is positive.

This is called a "sign analysis".

$x=-6$: $(-6-3)(-6+5) = (-)(-) = + > 0$ good

$x=0$: $(0-3)(0+5) = (-)(+) = - < 0$ not good

$x=4$: $(4-3)(4+5) = (+)(+) = + > 0$ good

So the soln set 

is $(-\infty, -5] \cup [3, \infty)$

2d) $f(x) = \dfrac{x^2+2}{2x+1}$ Dom: $2x+1 \neq 0$ so $\boxed{x \neq -\frac{1}{2}}$

Roots: $x^2+2=0$ no soln, no roots

e) $f(x) = \dfrac{x^2+3x}{x}$ Dom: $x \neq 0$

Roots: $x^2+3x=0$

$x(x+3)=0$, $x = \cancel{0}, -3$

discard (not in domain)

$x=-3$ is only root

f) $f(x) = \sqrt[3]{\dfrac{x-2}{x+6}}$ Only one domain restriction, $\boxed{x \neq -6}$

For any root that is odd, the radicand can be positive or negative. e.g. $\sqrt[3]{-8} = -2$

So $\dfrac{x-2}{x+6}$ can be < 0 or > 0.

Dom: $x \neq -6$

Roots: $\sqrt[3]{\dfrac{x-2}{x+6}} = 0$ when $x-2=0$

or $\boxed{x=2}$

3) $f(x) = \begin{cases} 2x+2, & \text{if } x < 1 \\ 4-x, & \text{if } 1 < x < 3 \\ \dfrac{3+x}{3-x}, & \text{if } x > 3 \end{cases}$

$f(0)$ does not exist, since $0 \notin \text{dom } f$.

$f(1)$ " " " " $1 \notin \text{dom } f$.

$f(5) = \dfrac{3+5}{3-5} = \dfrac{8}{-2} = -4$

Roots? Where is $f(x) = 0$?

$2x + 2 = 0$ at $x = -1$, so yes, it has a root.

The other two pieces give values that are not in the portion of the domain where they live.

Skipping to #8.

8) $32^{4/5} = (32^{1/5})^4 = 2^4 = 16$

9) $17^0 = 1$, $8^{-1/3} = \dfrac{1}{8^{1/3}} = \dfrac{1}{2}$, $4^{3/2} = \left(4^{1/2}\right)^3 = 8$,

$100^{\frac{1}{2}} - 64^{\frac{1}{2}} = 10 - 8 = 2$,

$(100 - 64)^{1/2} = 36^{1/2} = 6$,

$-3^2 = -(3^2) = -9$, $\sqrt{25} = 5$, $\sqrt{-9}$ undefined.

11) $x^{1/3} = \sqrt[3]{x}$, $-x^{1/2} = -\sqrt{x}$, $(-x)^{1/2} = \sqrt{-x}$.

$x^{9/5} = \sqrt[5]{x^9}$ or $(\sqrt[5]{x})^9$, $-3x^{2/3} = -3\sqrt[3]{x^2}$,

$2(xy)^{-3/4} = 2/(xy)^{3/4} = \dfrac{2}{\sqrt[4]{x^3y^3}}$

12. $\sqrt{x}$, $x \geq 0$ $\dfrac{1}{\sqrt{x}}$, $x > 0$

$\sqrt{-x}$, $x \leq 0$ $\sqrt{x-6}$, $x \geq 6$

$\sqrt{x^2}$, $x \in \mathbb{R}$ $\sqrt{6-x}$, $x \leq 6$

$\sqrt[3]{x}$, $x \in \mathbb{R}$

$\cancel{13. \sqrt{(-x)^5} = \sqrt{\underbrace{(-x)(-x)}_{x}\underbrace{(-x)(-x)}_{x}(-x)} = \sqrt{\ }}$

13. • $\sqrt{(-x)^5} = \sqrt{(-1)^5(x^5)} = \sqrt{(-1)^5}\sqrt{x^5} = \sqrt{-x^5}$ f

• $\sqrt{(-x)^5} = ((-x)^5)^{1/2} = (-x)^{5/2}$ b

• $\sqrt{(-x)^5} = (\sqrt{-x})^5$ g

14. a) $(x^{1/2})^{-1/3} = x^{(1/2)(-1/3)} = x^{-1/6} = \dfrac{1}{x^{1/6}}$

b) $\left(\dfrac{3x}{y}\right)^{-2} = \left(\dfrac{y}{3x}\right)^2 = \dfrac{y^2}{9x^2}$

c) $x^{1/2}x^{-2/3} = x^{1/2-2/3} = x^{3/6-4/6} = x^{-1/6}$

$= \dfrac{1}{x^{1/6}}$

14d) $\sqrt{x^{-7}} = (x^{-7})^{1/2} = x^{-7/2}$

e) $\left(\frac{a^{-2}}{b^{-2}} + \frac{b^{-2}}{a^{-1}}\right)^{-1} = \left(\frac{b^2}{a^2} + \frac{a}{b^2}\right)^{-1}$

$= \left(\frac{b^4 + a^3}{a^2b^2}\right)^{-1} = \frac{a^2b^2}{b^4+a^3}$