

2d) $f(x,y) = x+y-x^2-y^2$, $\underbrace{x+2y=6}_{g(x,y)=c}$
 Let's do the video way; keep $x+2y=6$ as is.

$$\frac{\partial f}{\partial x} = \lambda \frac{\partial g}{\partial x} \quad \longrightarrow \quad 1-2x = \lambda \cdot 1 \quad (1)$$

$$\frac{\partial f}{\partial y} = \lambda \frac{\partial g}{\partial y} \quad \longrightarrow \quad 1-2y = \lambda \cdot 2 \quad (2)$$

$$g(x,y) = c \quad \longrightarrow \quad x+2y = 6 \quad (3)$$

Eliminate λ from first two:

$$(1) \quad 1-2x = \lambda \quad \xrightarrow[\text{into } (2)]{\text{sub } \lambda} \quad 1-2y = (1-2x) \cdot 2$$

Solve for
 y in terms
 of x

$$-2y = 2 - 4x - 1$$

$$-2y = 1 - 4x$$

$$\boxed{y = \frac{1-4x}{-2}}$$

Sub y
 into

constraint

$$x+2y=6$$

$$x + 2 \left(\frac{1-4x}{-2} \right) = 6$$

$$-2x + 2 - 8x = -12$$

$$-10x = -14$$

$$x = 14/10 = 7/5 = x$$

sub x
 into
 constraint

$$\frac{7}{5} + 2y = 6$$

$$\boxed{y = \frac{23}{10}}$$