2c. This one is a bit different, since if you eliminate λ, you get ~~xxx~~ no x or y either! Shown below using book's way:

$$f(x,y) = x^2 - y^2 \ ; \qquad x^2+y^2 = 4 \rightarrow g(x,y) = x^2+y^2-4$$

$$F = x^2 - y^2 + \lambda(x^2+y^2-4)$$

$$F_x = 2x + 2\lambda x = 0 \longrightarrow 2x(1+\lambda) = 0$$
$$F_y = -2y + 2\lambda y = 0 \longrightarrow -2y(1-\lambda) = 0$$
$$F_\lambda = x^2+y^2-4 = 0 \longrightarrow x^2+y^2-4 = 0$$

Consider F_x and F_y. They basically say $AB=0$ so either $A=0$ or $B=0$. That is,

$$F_x = 2x(1+\lambda) = 0 \longrightarrow 2x = 0 \ \boxed{x=0} \quad \text{or} \quad 1+\lambda = 0 \ \boxed{\lambda = -1}$$

$\lambda = -1$ isn't relevant to the constraint, but substituting $x=0$ into $x^2+y^2=4$ gives two possible y values: $0+y^2 = 4 \rightarrow \boxed{y = \pm 2}$

Test the two points $(0,2)$ and $(0,-2)$ into $f(x,y)$ to see which gives the maximum.