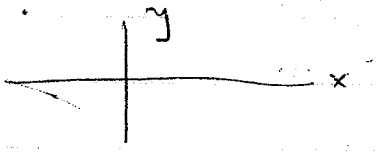
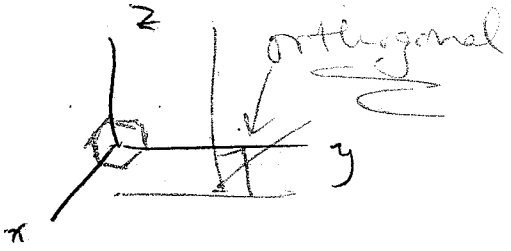
Sec 25 – "3-Space"



2-space



3-space

Equations of line & plane, respectively.

$px + qy + r = 0$ (notice: slope $= -\frac{p}{q}$)

$px + qy + rz + s = 0$

We're used to $y = mx + b$, but when we try to visualize eqns. of planes in 3-space, it's better to jump off from the alternate form here: $px + qy + r = 0$

We could use this form to find the eqn. of a line in 2-space given 2 pts (x_1, y_1) & (x_2, y_2) by solving the system

System to be solved simultaneously →

$$\begin{cases} px_1 + qy_1 + r = 0 \\ px_2 + qy_2 + r = 0 \end{cases}$$

We solve for $p + q$

(r resolves when we get $p + q$)

Likewise, if we have 3 points in 3-space (two are not coplanar with the third), solving the system:

$$\begin{aligned} px_1 + qy_1 + rz_1 + s &= 0 \\ px_2 + qy_2 + rz_2 + s &= 0 \\ px_3 + qy_3 + rz_3 + s &= 0 \end{aligned}$$

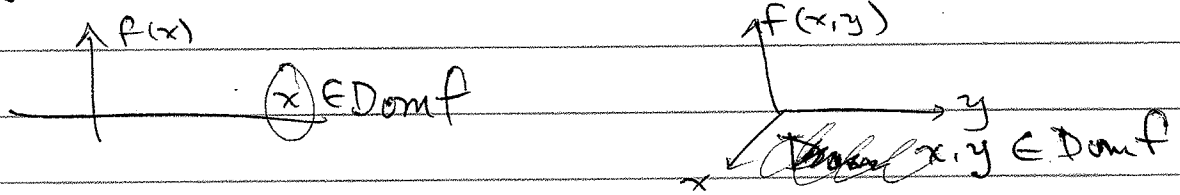
will give us the eqn. of the plane through those 3 pts:

We don't solve for x, y, z, but for p, q, r.

Sec 2.6 Functions of Two Variables

Note: Errors in text: p. 208 $g(x) = \begin{cases} 1 \\ 0 \end{cases}$
should read $g(x,y) = \dots$
p. 210 graph (a) should say $y \geq 0$ under it, not $y \geq x$.

$y = f(x) \longrightarrow z = f(x,y)$

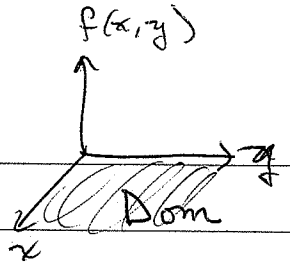


We won't try too hard to visualize the graphs. Geometrically, they are interesting & important, but for our needs, we can still imagine the (x,y)-coordinate system, but now the domain encompasses restrictions on both x & y.

For example: $f(x,y) = x \cdot \sqrt{y}$ (not linear) has domain $\{(x,y) : y \geq 0\}$ "the set of (x,y) such that $y \geq 0$; $x \in \mathbb{R}$"

For multivariable fcns, it is best to represent the domain in set notation, as seen.

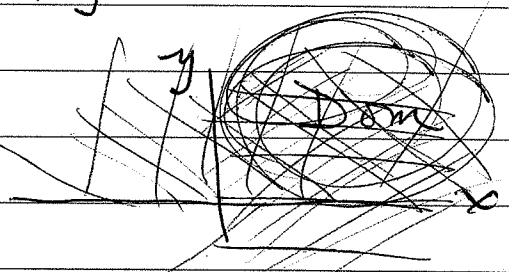
We can also graph the domain; but stick to 2-space and imagine the values $f(x,y)$ lie above, below, or on the (x,y)-plane.



Here is a sketch of the domain $\{(x,y) : x \geq 0, y \geq 0\}$

e.g. $f(x,y) = \sqrt{xy}$

In 2-space, we'd draw



That is, the domain is the first quadrant and here it includes the axes there.

Look at Ex 26.1 on pp 209-210. The words "SHADE" is used instead of showing cross-hatching.

Digression Sketch these

$y = \sqrt{x}$ $\quad y = \frac{1}{x}$ $\quad y = \ln x$

2-Dim