Sec 2.4 HW – extra – selected problems

#1 (a) $\sum_{i=1}^{5} \sqrt{i} = \sqrt{1} + \sqrt{2} + \sqrt{3} + \sqrt{4} + \sqrt{5}$

(d) $\sum_{i=0}^{n-1} (-1)^i = (-1)^0 + (-1)^1 + (-1)^2 + \dots + (-1)^{n-1}$

$= \cancel{1} - \cancel{1} + \cancel{1} - \cancel{1} + \dots + (-1)^{n-1}$

The answer will be either 0 or 1, depending on whether n is ~~odd~~ even (the terms cancel ~~cancel~~) or odd (a leftover $(-1)^{\text{even}} = 1$)

#3 c) Write the equivalent sum where $m=0$ is lower bound:

$$\sum_{i=6}^{10} (-4i+10)^2 = \sum_{m=0}^{4} [-4(i+6)+10]^2$$

We subtracted 6 from i & n and added it to the a_i term. This produces equivalent sums

Do the same for $m = 5$ as lower bound:

$$\sum_{i=6}^{10} \longrightarrow \sum_{m=5}^{9} [-4(i+1)+10]^2$$

#4) Find the numeric value – this means use the formulas + other properties:

a) $\sum_{i=4}^{8}(3i-2)$ We could do this w/o formula.

$$\sum = (3(4)-2) + (3(5)-2) + (3(6)-2) + (3(7)-2) + (3(8)-2) = 80$$

Or like this:

$$\sum_{i=4}^{8}(3i-2) = 3\sum_{i=4}^{8} i - \sum_{i=4}^{8} 2$$

Change the index so you can use Gauss' formula

$$\sum = 3\sum_{i=1}^{5}(i+3) - \sum_{i=1}^{5} 2$$

doesn't affect the 2

$$= 3\sum_{i=1}^{5} i + 3\sum_{i=1}^{5} 3 - \sum_{i=1}^{5} 2$$

$$= 3\frac{(5)(\cancel{6})}{\cancel{2}} + 3(3)(5) - (2)(5)$$

$$= 45 + 45 - 10 = 80$$

Was probably easier to use other method :(

#4f) $\sum_{i=1}^{5} i^3$ = from formula $\left(\frac{n(n+1)}{2}\right)^2$

$$\left(\frac{5(6)}{2}\right)^2 = 15^2 = 225$$

which is faster than $1^3 + 2^3 + 3^3 + 4^3 + 5^3$

$$\begin{array}{r} 36 \\ 64 \\ \hline 100 \end{array}$$

#4h) $\sum_{i=20}^{75} i$ Either re-index as before or use the subtraction idea

$$\sum_{i=20}^{75} i = \sum_{i=1}^{75} i - \sum_{i=1}^{19} i$$

by Gauss formula $= \frac{75(76)}{2} - \frac{19(20)}{2}$

$$= (75)(38) - (19)(10) \quad \text{etc.}$$

#4k) $\sum_{i=1}^{20} (i+1)(i-1)$ Expand the summand + apply formulas!

$$\sum_{i=1}^{20} (i^2 - 1) = \sum_{i=1}^{20} i^2 - \sum_{i=1}^{20} 1$$

= from formula for $\sum i^2 = \frac{n(n+1)(2n+1)}{6}$ $\longrightarrow$

Rewrote it $\sum_{i=1}^{20} i^2 - \sum_{i=1}^{20} 1$

Apply $\sum_{i=1}^{n} i^2$ $= \dfrac{\overset{10}{\cancel{20}}(\overset{7}{\cancel{21}})(2(20)+1)}{\cancel{6}\,\cancel{2}} - 20(1)$

$= (10)(7)(41) - (20) = 2870 - 20$

$= 2850$

$\begin{array}{r} 41 \\ \underline{70} \\ 2870 \end{array}$

#7, Rewrite in terms of n (you don't have value of n, so you can't simplify to a number)

d) $\sum_{i=1}^{n} (i+1)(i+2)$ Expand so you can see each term & which formula to apply

$\downarrow$

$\sum_{i=1}^{n} (i^2 + 3i + 2) = \sum_{i=1}^{n} i^2 + 3\sum_{i=1}^{n} i + \sum_{i=1}^{n} 2$

$= \boxed{\dfrac{n(n+1)(2n+1)}{6} + 3\left(\dfrac{n(n+1)}{2}\right) + 2n}$