Sec 2.2

#1 $x^2 + 8x + \underline{16} = (x+4)^2$

from $\left(\frac{1}{2} \text{ of } 8\right)^2 = 4^2 = 16$

$x^2 - 20x + \underline{100}$ $\quad \frac{1}{2}$ of $(-20) = -10$

$= (x-10)^2$ $\quad (-10)^2 = 100$

$x^2 + 7x + \underline{\left(\frac{7}{2}\right)^2} = x^2 + 7x + \frac{49}{4} = \left(x + \frac{7}{2}\right)^2$

Use fractions or $\sqrt{}$ when those appear in final answer

Last 2 special products:

- square of a sum $(a+b)^2$
- square of a difference $(a-b)^2$

In general: $x^2 + bx + \left(\frac{b}{2}\right)^2 \stackrel{?}{=} \left(x + \frac{b}{2}\right)^2$

FOIL $\left(x + \frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)\left(x + \frac{b}{2}\right)$

$= x^2 + \frac{b}{2}x + \frac{b}{2}x + \frac{b}{2} \cdot \frac{b}{2}$

$= x^2 + \frac{2b}{2}x + \left(\frac{b}{2}\right)^2$

$= x^2 + bx + \left(\frac{b}{2}\right)^2$ ✓

$x^2 + 8x - 11 = (x + \square)^2 + \text{const}$

$= x^2 + 8x + (4)^2 - 16 - 11 = (x+4)^2 - 27$

So far $x^2 + bx + c$

But $ax^2 + bx + c$

$a \neq 1$

# 3e $4x^2 + 24x + 3$

$= 4(x^2 + 6x) + 3$

$= 4(x^2 + 6x + 9) + 3$

$\frac{1}{2} \cdot 6 = 3$, $3^2 = 9$ $\quad (4)(9) = 36$

$= 4(x+3)^2 - 36 + 3$ — const

$= \boxed{4(x+3)^2 - 33}$ Done

If just $x^2 + 6x + 3$

$= x^2 + 6x + 9 - 9 + 3$

$= (x+3)^2 - 6$

In general $ax^2 + bx + c$

$= a(x^2 + \frac{b}{a}x) + c$

$= a(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2}) - \frac{b^2}{4a^2} + c$

$(\frac{1}{2} \text{ of } \frac{b}{a})^2 = (\frac{1}{2} \cdot \frac{b}{a})^2 = (\frac{b}{2a})^2$