Sec 19, 20 Polynomials + roots; Rational fns

Sec 19 Polynomials (mostly) + root fns.

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$$

These fcns are naturals for investigating extremes, concavity, and points of inflection because higher degree ones have (can have) many factors, and their derivatives do, too.

ex (from HW p. 153 #2c)

$$h(x) = \frac{3}{2}x^4 - 2x^3 - 6x^2 + 8$$

$$h'(x) = 6x^3 - 6x^2 - 12x$$
$$= 6x(x^2 - x - 2)$$
$$= 6x(x-2)(x+1)$$

Crit values

$$h'(x) = 0 \text{ at } \underbrace{\boxed{x = 0, 2, -1}}_{\text{Critical points}}$$

By FDT we can inspect where f is increasing, decreasing, ~~etc.~~ has POI. But I tend to go to the SDT + see the sign of $h''(c)$ to determine whether the fcn has a max, min or neither at c.

②

Notice, also, about polynomials, that there is no instances where the derivative DNE. Derivatives of polynomials are polynomials. Domain $= \mathbb{R}$ for $f', f'', \ldots$

$h''(x) = ?$ Go back to the unfactored $h'(x)$:

$$h'(x) = 6x^3 - 6x^2 - 12x$$
$$h''(x) = 18x^2 - 12x - 12$$

Check crit values into $h''(x)$:

$h''(0) = -12 < 0 \rightarrow h(0)$ is a max

$h''(2) = 36 > 0 \rightarrow h(2)$ is a min

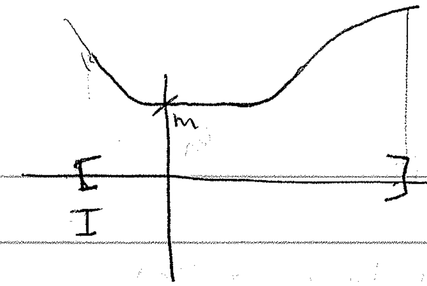
$h''(-1) = 12 > 0 \rightarrow h(-1)$ is a min

Where does $h''(x) = 0$? Factor it to see.

$$\begin{aligned} h''(x) &= 18x^2 - 12x - 12 \\ &= 6(3x^2 - 2x - 2) \\ &= 6(3x+2)(x-1) = 0 \end{aligned}$$

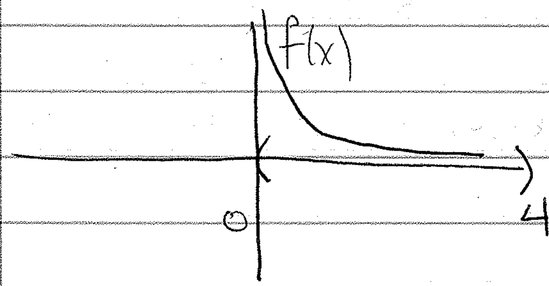
So $x = -\frac{2}{3}$ and $x = 1$ are pts of inflec

(Why are we certain

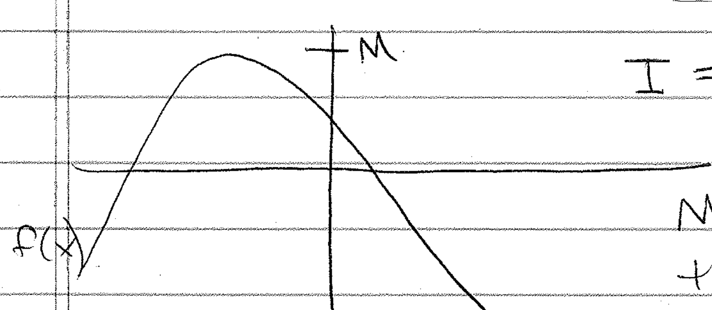


$f(x) \geq m$
for all $x \in I$

What happens when we let ${}_{\wedge}^{\text{interval}}$ I be open?
The short answer is that the endpts become an issue. But it's still a valid assumption.



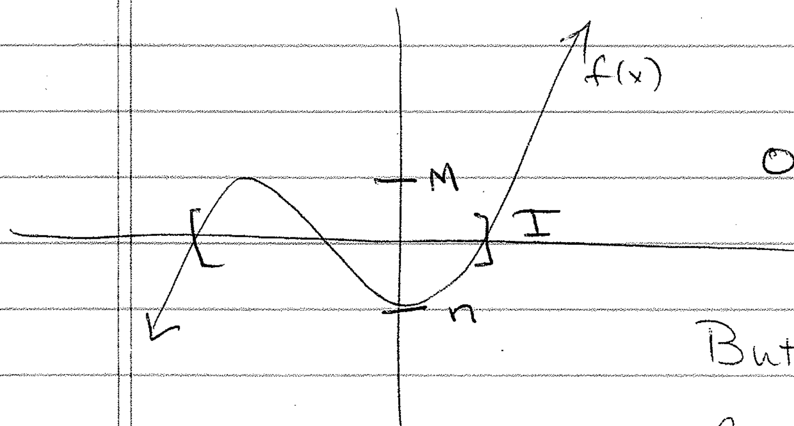
Suppose $I = (0, 4)$ $\longrightarrow$ $f(x) = \frac{1}{x}$ has no abs max M or abs min m in I.



$I = \mathbb{R}^*$ (the entire number line)

M is the abs max, but there is no abs min because $f(x)$ is unbounded in the negative sense.

* If I = a smaller open interval, will $f(x)$ have an absolute minimum?



On I, $f(x)$ has a ${}_{\wedge}^{\text{abs}}$ max M and an abs min m.

But on $\mathbb{R}$, it has neither an abs max or min.

④

The approach to finding **absolute** extremes is to first check the value of the fcn at any **local** extremes (i.e., where $f'(x)=0$ or DNE, the critical pts), **then** to check $f(x)$ at its endpts, if it has any.

Ex 22.3 p. 182 $\quad f(x) = x^3 - 3x$

Check for absolute extrema on the intervals