5

Ex 18.4 $\quad f(x) = x^3 + 2x^2 + x + 6$ $\quad$ Dom: $\mathbb{R}$

Crit values

$f'(x) = 3x^2 + 4x + 1 = (3x+1)(x+1)$ $\quad$ $f(0) = 6$

$f'(x) = 0$ at $x = -\frac{1}{3}, -1$ $\quad$ $f''(x) = 6x + 4$

Instead of using first derivative test, go straight to second derivative if you want to check concavity at c #5

$f''(-\frac{1}{3}) = -2 + 4 > 0$ c. up

$f''(-1) = -6 + 4 < 0$ c. down

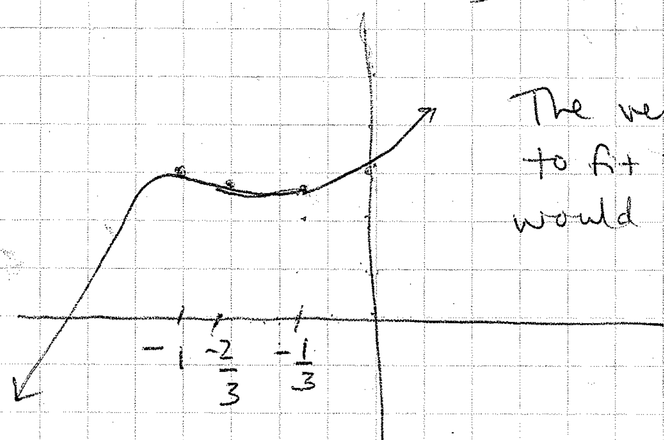
So at $x = -\frac{1}{3}$ has a local min: $f(-\frac{1}{3}) = 5\frac{23}{27}$

and at $x = -1$, it has a local min: $f(-1) = 6$

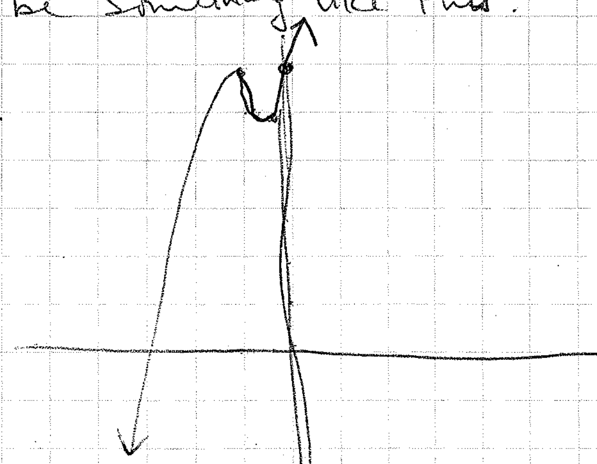
Inflection pt? $\quad f''(x) = 6x + 4 = 0$ at $x = -\frac{4}{6}$ or $-\frac{2}{3}$

Notice this is midway between $x = -1$ and $-\frac{2}{3}$

$f(-\frac{2}{3}) = (-\frac{2}{3})^3 + 2(-\frac{2}{3})^2 - \frac{2}{3} + 6 = \frac{-8}{27} + \frac{8}{9} - \frac{2}{3} + 6 = 5\frac{25}{27}$



The vertical scale is compressed to fit the range. Actual graph would be something like this:



In class example $f(x) = \frac{1}{3}x^3 + \frac{1}{2}x^2 + x$

Range, Dom: $\mathbb{R}$ (poly)

x-int: $(0,0)$

y-int: $(0,0)$

Crit pts: none

Inflection pt. $x = -1/2$, $y = ?$

Intervals of ↑, ↓

f increasing on $(-\infty, \infty)$

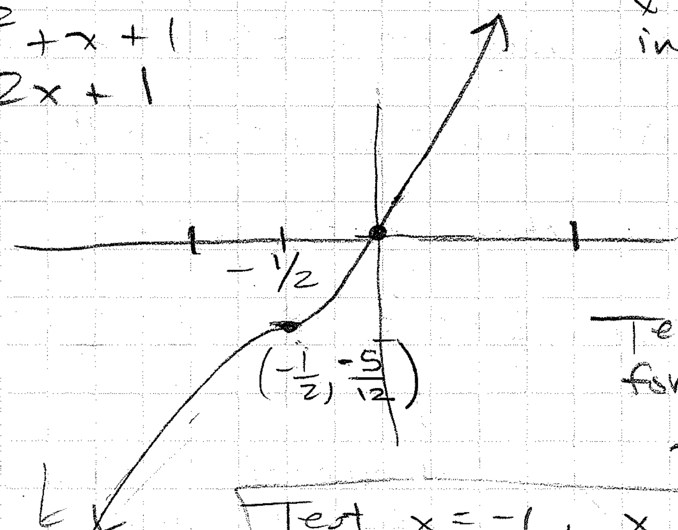
$\mathbb{R}$

Since $f''(-\frac{1}{2}) = 0$, there is no max/min at $x = -1/2$; rather, it's an inflection

$f' = x^2 + x + 1$

$f'' = 2x + 1$

$f(-\frac{1}{2}) = -5/12$



Test in terms of intervals for concavity:

$x = -1/2$ is inflection pt

To get an accurate picture of concavity you must test $f''(x)$

Test $x = -1$, $x = 0$ into $f''(-1) = -2 + 1 = -1 < 0$, concave down

$f''(0) = 2(0) + 1 = 1 > 0$, concave up

$$f(-1/2) = \frac{1}{3}\left(-\frac{1}{2}\right)^3 + \frac{1}{2}\left(-\frac{1}{2}\right)^2 + -\frac{1}{2}$$

$$= -\frac{1}{24} + \frac{1}{8} - \frac{1}{2} = -\frac{5}{12}$$

LCD 24

On $(-\infty, -\frac{1}{2})$ concave down

On $(-\frac{1}{2}, \infty)$ concave up

(7) <u>Ex 18.5</u> $f(x) = x - \ln x$ Dom $x > 0$ (no y-int)

Roots? $x - \ln x = 0 \rightarrow x = \ln x$

There is no x where $x = \ln x$. We know because the graphs of $y = x$ and $y = \ln x$ do not intersect. (You don't have to analyze it that far, but since we like to determine roots if we can, I give it here)

Find critical values:

$f'(x) = 1 - \frac{1}{x}$ DNE at $\boxed{x = 0}$

$1 - \frac{1}{x} = 0$ at $\boxed{x = 1}$

By first der. test we know $f \downarrow$ on $(0, 1)$ and $\uparrow$ on $(1, \infty)$, but if we go straight to second der. test with $f''(c)$ we'll get the same info:

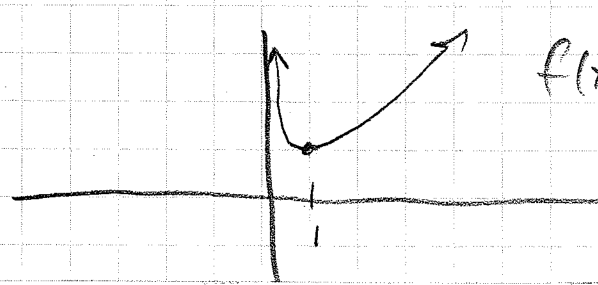
$f''(x) = \frac{1}{x^2}$, $f''(1) = 1 > 0$, so conc. up + $x = 1$ is local min

(We will still have to show test values of f' for many problems, so here I'll show it:

$f'(\frac{1}{2}) = 1 - \frac{1}{1/2} = -1 < 0$, $f \downarrow (0,1)$; $f'(\frac{3}{2}) = 1 - \frac{1}{3/2} = \frac{1}{3} > 0$, $f \uparrow (1, \infty)$

There is no POI. $f'' > 0$ for all x, hence concave up on $(0, \infty)$ with minimum

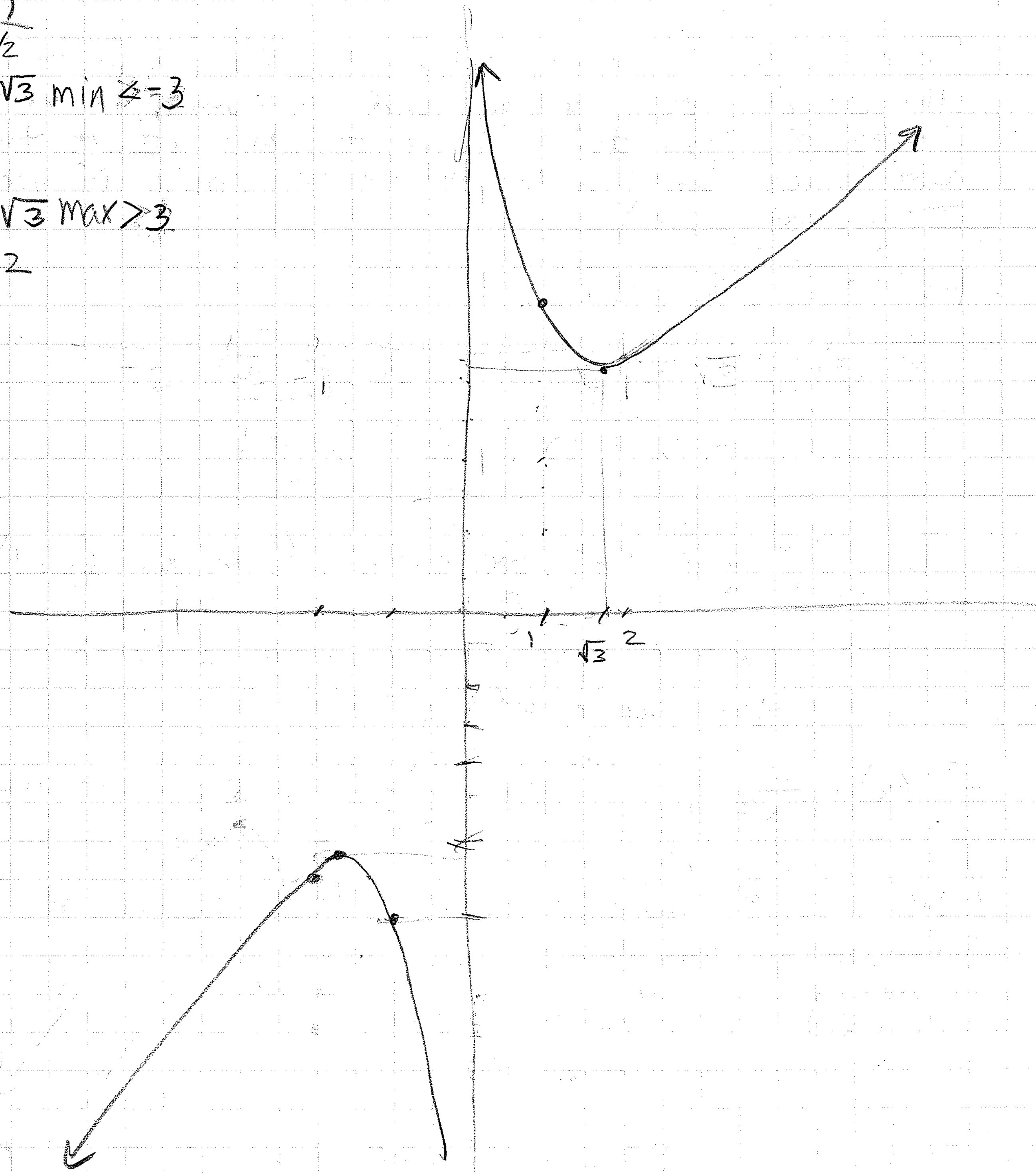
$f(1) = 1 - \ln 1 = 1 - 0 = 1$



$f(x) = x - \ln x$

9) Graph of $f(x) = x + \frac{3}{x}$

x	$f(x)$
-2	$-3\frac{1}{2}$
$-\sqrt{3}$	$-6/\sqrt{3}$ min ≤ -3
-1	-4
1	4
$\sqrt{3}$	$6/\sqrt{3}$ max > 3
2	$3\frac{1}{2}$



(8)

2e) $f(x) = x + \frac{3}{x} = 0$ $x \neq 0$, no roots $\frac{x^2+3}{x} \neq 0$

$f'(x) = 1 - 3/x^2$ $x = 0$ crit value

set $f'(x) = 0$, $1 - 3/x^2 = 0$

$f''(x) = 6/x^3$ $1 = 3/x^2$, $x = \pm\sqrt{3}$

$f''(c_1) > 0$ at $x = \sqrt{3}$ c. up local min

$f''(c_2) < 0$ at $x = -\sqrt{3}$ c. down local max

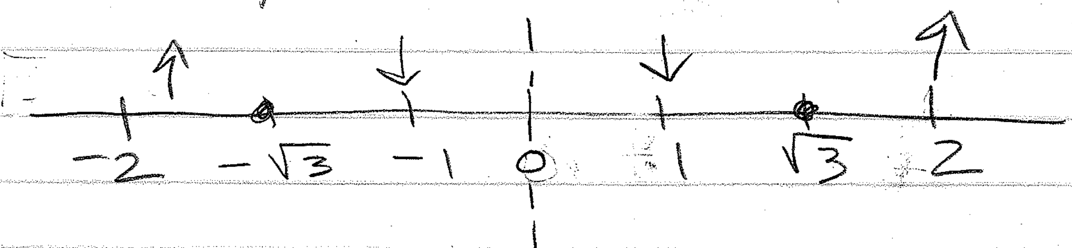
$f''(x) = \frac{6}{x^3} \neq 0$ No POI

Calculate $f(x)$ $f(c_1) = f(\sqrt{3}) = \sqrt{3} + \frac{3}{\sqrt{3}} = \frac{3+3}{\sqrt{3}} = \frac{6}{\sqrt{3}}$

$f(c_2) = f(-\sqrt{3}) = -\sqrt{3} + \frac{3}{-\sqrt{3}} = \frac{-3+3}{-\sqrt{3}} = \frac{-6}{\sqrt{3}}$

So, we know extremes $(\sqrt{3}, 6/\sqrt{3})$, $(-\sqrt{3}, -6/\sqrt{3})$ and that $x = 0$ is a VA.

We could actually sketch this without the first der. test for intervals of ↑ and ↓ but let's practice that skill:



$f'(-2) = 1 - 3/4 > 0$ $f'(1) = 1 - 3/1 < 0$

$f'(-1) = 1 - 3/1 < 0$ $f'(2) = 1 - 3/4 > 0$

We might as well calculate the fcn at these values, since we'll get a better graph