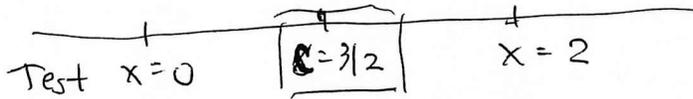




2a.  $y = 2 + 24x - 8x^2$ ,  $y' = 24 - 16x = 0$ ,  $\left| \begin{array}{l} x = 3/2 \\ \text{(call it "c" for critical point)} \end{array} \right|$



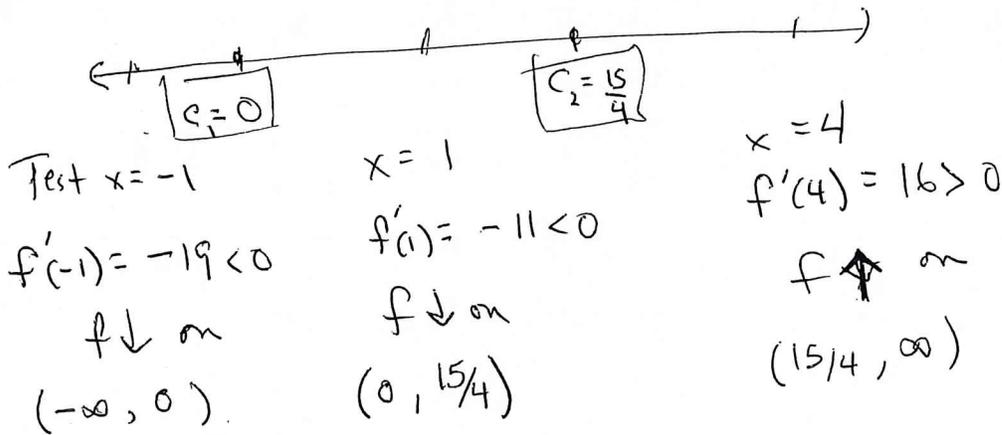
$f'(0) = 24 > 0$   
 $f \uparrow$  on  $(-\infty, 3/2)$

$f'(2) = -8$   
 $f \downarrow$  on  $(3/2, \infty)$

$f(c)$  is a local max

The value of the fun at  $c = 3/2$  is  $f(3/2) = 2 + 36 - 18 = 20$   
 So  $(3/2, 20)$  is a local max.

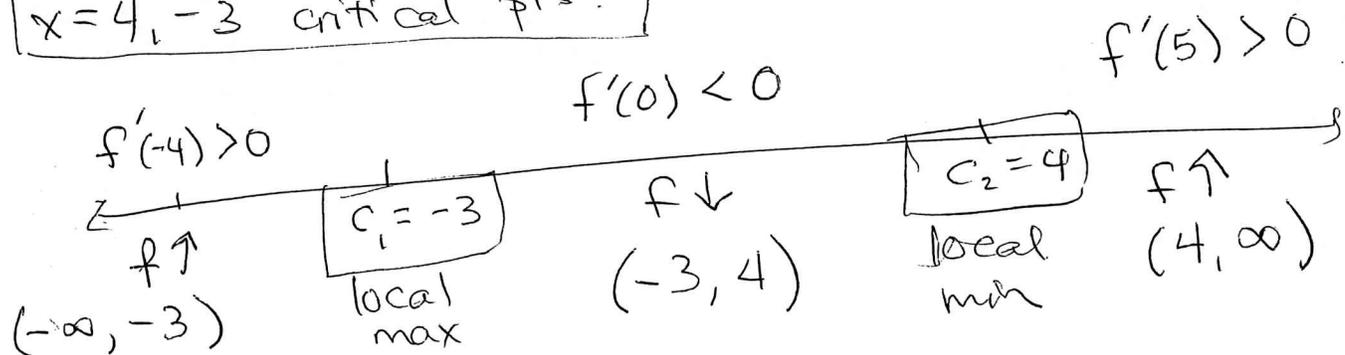
2b.  $f(x) = x^4 - 5x^3 + 100$ ,  $f'(x) = 4x^3 - 15x^2 = 0$   
 $x^2(4x - 15) = 0$   
 $x = 0, 15/4$  critical points



$f(0)$  neither max nor min  
 $f(15/4)$  local min

2c.  $f(x) = \frac{2}{3}x^3 - x^2 - 24x - 10$ ,  $f'(x) = 2x^2 - 2x - 24 = 0$   
 Reduce + factor:  $x^2 - \frac{1}{2}x - 12 = (x-4)(x+3) = 0$

$x = 4, -3$  critical pts.



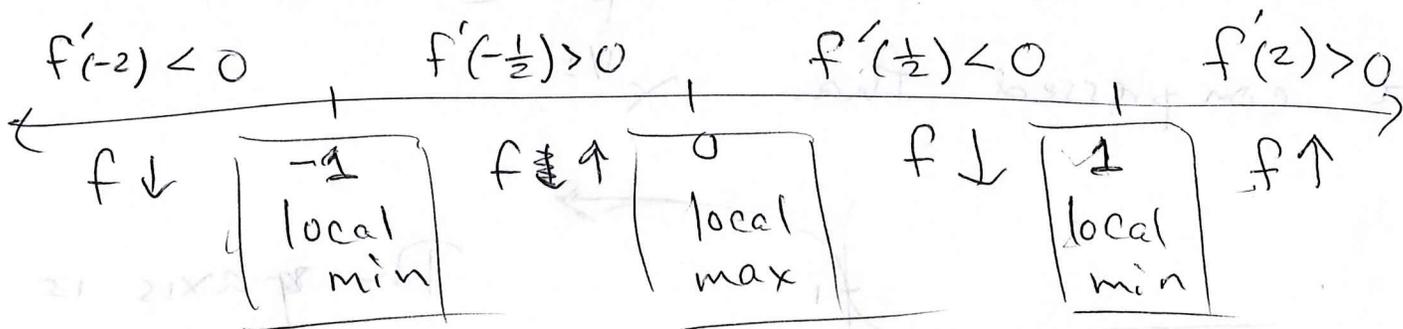
(2d)  $f(x) = (x^2 - 1)^8$ ,  $f'(x) = 8(x^2 - 1)^7 \cdot 2x = 16x(x^2 - 1)^7$

$\rightarrow 16x(x^2 - 1)^7 = 0$ ,  $16x = 0$  or  $(x^2 - 1)^7 = 0$

That is,  $x = 0$  or  $x^2 - 1 = 0$ ,  $x = \pm 1$

Crit pts.  $x = 0, 1, -1$

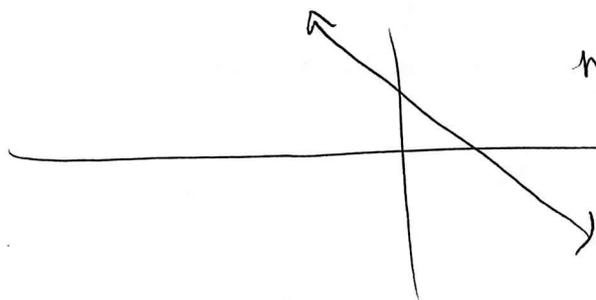
Unfortunately, the test values will include fractions  $\pm \frac{1}{2}$ .



$f \uparrow$  on  $(-1, 0) \cup (1, \infty)$ ;  $f \downarrow$  on  $(-\infty, -1) \cup (0, 1)$

(2e)  $y = -2x + 4$ ,  $y' = \text{slope} = -2 < 0$

$f \downarrow$  on  $(-\infty, \infty)$



no extremes!

2f.

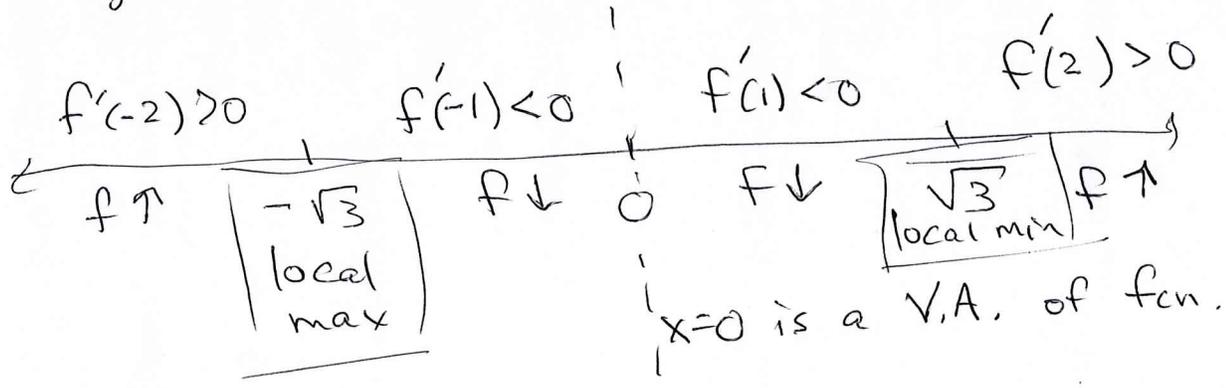
$$f(x) = x + \frac{3}{x}$$

Take note of domain:  
 $x \neq 0$

$f'(x) = 1 - \frac{3}{x^2} = 0$  while it's true that  $f'(0)$  DNE, since  $f(0)$  is not a value of the fcn,  $x=0$  is not a critical point.

$1 = \frac{3}{x^2} \rightarrow x^2 = 3$   
Critical pts.  $\rightarrow$   $x = \pm \sqrt{3}$

However, it is an interesting point, so we'll include it when we partition the number line doing the FDT.



$f \uparrow$  on  $(-\infty, -\sqrt{3}) \cup (\sqrt{3}, \infty)$ ;  $f \downarrow$  on  $(-\sqrt{3}, 0) \cup (0, \sqrt{3})$

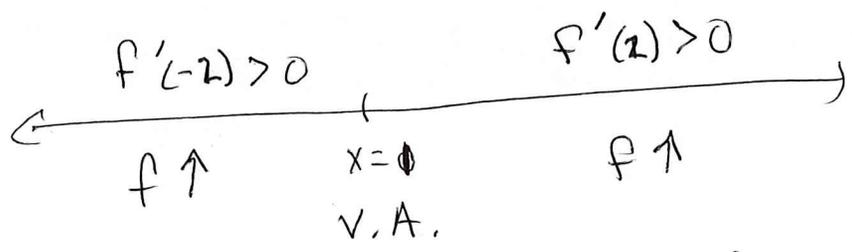
2g.

$$f(x) = \frac{x-2}{x-1}$$

Dom:  $x \neq 1$  (so even if  $f'(0)$  DNE, it's not a crit. pt. since 0 is not in the domain of  $f$ )

$$f'(x) = \frac{(1)(x-1) - (x-2)(1)}{(x-1)^2} = \frac{1}{(x-1)^2}$$

where, again,  $x=1$  is not in the dom, and  $f' \neq 0$  anywhere, but still, inspect left and right of  $x=0$ ,



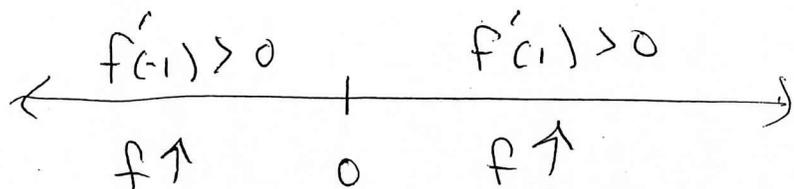
No extrema.  $f \uparrow$  on  $(-\infty, 1) \cup (1, \infty)$

(2h.)

$$f(x) = 1 + x^{1/5}, \quad \text{Dom. } x \in \mathbb{R}$$

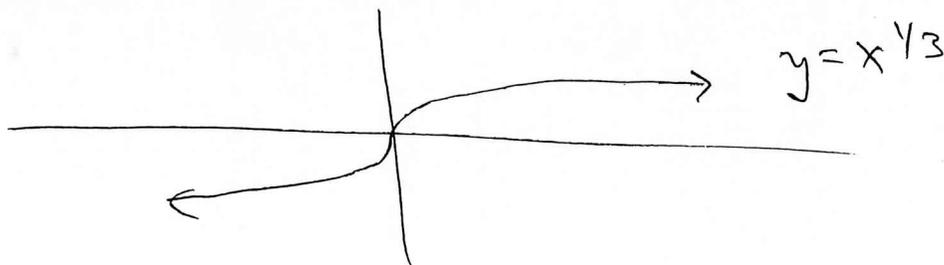
$$f'(x) = \frac{1}{5} x^{-4/5} = \frac{1}{5x^{4/5}}$$

$x=0$  is crit pt, since  $f'(0)$  DNE and 0 is in dom. of  $f$ .



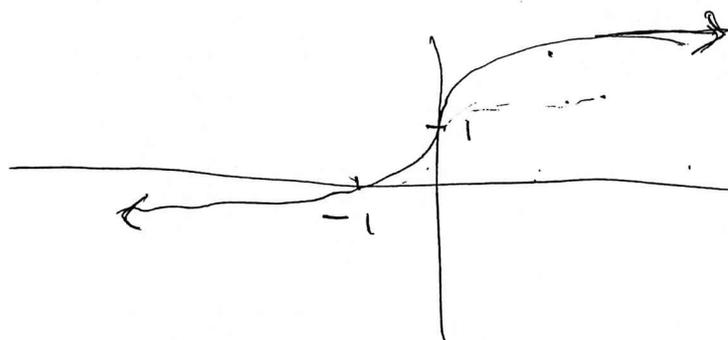
$f$  increases on  $(-\infty, \infty)$

At  $x=0$ , there's a vertical tangent. If we recall the odd root fun  $y = x^{1/3}$ , we can see the idea.



Any odd root fun  $y = x^{1/n}$  has this shape.

$y = 1 + x^{1/5}$  is moved up 1 unit, but more compressed than  $x^{1/3}$ .



The  $y$  axis is the vertical tangent.

$f$  is increasing everywhere, but we'll see when we apply the second derivative test (SDT) that the sign of  $f''$  gives more information on  $f$ .

2k.

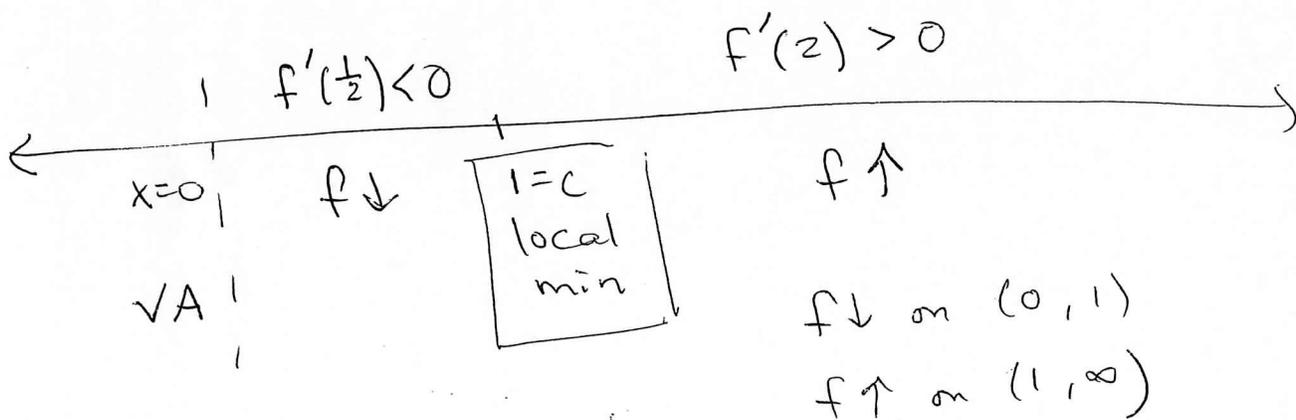
$f(x) = x - \ln x$  Domain  $x > 0$

$f'(x) = 1 - \frac{1}{x}$ ,  $f'(0)$  DNE, but  $f(x)$  is not defined on  $(-\infty, 0]$ , so we don't need to inspect left of zero.

$f'(x) = 1 - \frac{1}{x} = 0$

$\rightarrow 1 = \frac{1}{x} \rightarrow$

$x = 1$  critical pt.

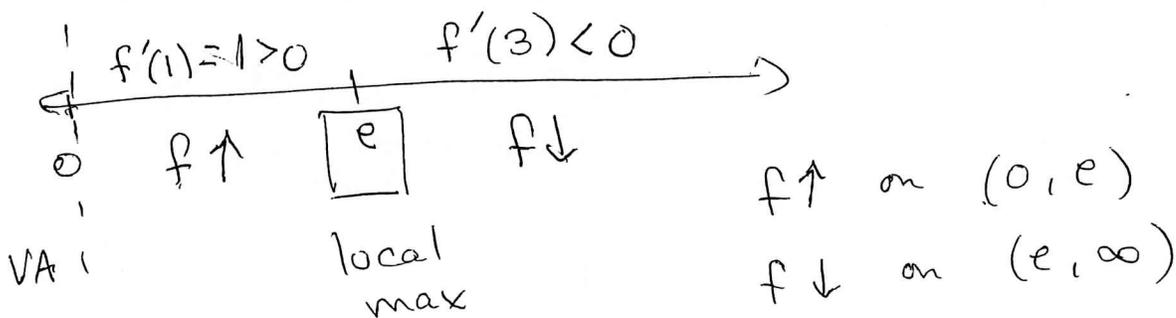


2l.

$f(x) = \frac{\ln x}{x}$ ,  $x > 0$  Domain.

$f'(x) = \frac{\frac{1}{x} \cdot x - \ln x \cdot 1}{x^2} = \frac{1 - \ln x}{x^2} = 0$

when  $1 - \ln x = 0 \rightarrow \ln x = 1 \rightarrow x = e$  crit point



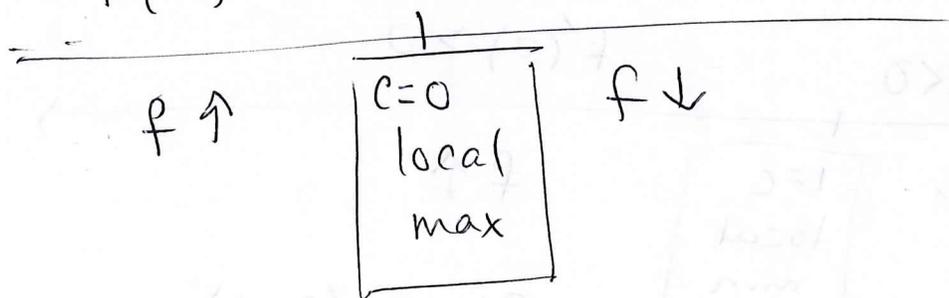
(2.m)

$$f(x) = x - e^x, \quad \text{Dom: } x \in \mathbb{R}$$

$$f'(x) = 1 - e^x \quad \text{Dom: } x \in \mathbb{R} \quad (\text{i.e. } f'(x) \text{ exists everywhere})$$

$$f'(x) = 1 - e^x = 0 \rightarrow 1 = e^x \rightarrow \boxed{\begin{array}{l} x = 0 \\ \text{crit pt} \end{array}}$$

$$f'(-1) > 0 \qquad f'(1) < 0$$



$f \uparrow$  on  $(-\infty, 0)$ ,  $f \downarrow$  on  $(0, \infty)$

(3.)

$$C'(x) = 6x^2 + 6x + 6 = 0$$

$$x^2 + x + 1 = 0 \quad \text{Does not factor!}$$

No critical points, No extremes. Is it increasing or decreasing everywhere? Quick inspection

of  $x$  negative, zero, + positive shows

$$x^2 + x + 1 > 0 \quad \text{for all } x, \text{ hence}$$

$f \uparrow$  on  $(-\infty, \infty)$ . Costs are rising

at all levels of production.