Sec 17 & 18 Example

For the fcn $f(x) = \frac{2}{3}x^3 - x^2 - 24x - 10$

find the following:

a) Critical pts:

Soln: Set $f'(x) = 0$

$$2x^2 - 2x - 24 = 0$$
$$x^2 - x - 12 = 0$$
$$(x-4)(x+3) = 0$$

Crit. #s: $x = 4, -3$

b) Intervals where $f(x)$ increases & decreases.

$\boxed{\text{FDT}}$ By the First Derivative Test we check values on either side of crit. #s.

c_1 c_2

Test Values: -4 $\boxed{-3}$ 0 $\boxed{4}$ 5

You can use $f' = x^2 - x - 12$, the reduced form of $f'(x)$.

Test	into $f(x) = x^2 - x - 12$
$x = -4$	$f'(-4) = 8 > 0$, so $f \uparrow$ on $(-\infty, -3)$
$x = 0$	$f'(0) = -12 < 0$, so $f \downarrow$ on $(-3, 4)$
$x = 5$	$f'(5) = 8 > 0$, so $f \uparrow$ on $(4, \infty)$

$f' > 0$, $f \uparrow$ $c = -3$ $f' < 0$ $f \downarrow$ $c = 4$ $f' > 0$, $f \uparrow$

c) Determine whether $c = -3$, $c = 4$ are local max, min.

Soln By FDT, since $f \uparrow$ left of $c = -3$ & $f \downarrow$ right of it,

$c = -3$ is a local max.

Alternately, by Second Derivative Test (SDT) you check whether $f''(c) > 0$ or < 0.

If $f''(c) > 0$, then f is concave up on interval containing c, so $f(c)$ is a local min.

If $f''(c) < 0$, then f is concave down on that interval, so $f(c)$ is local max.

Note What would we conclude if $f'(x) > 0$ left and right of the critical #?

Consider $f(x) = x^3$. $f'(x) = 3x^2$

$f' = 3x^2 = 0$ at $x = 0$. Testing $x = -1, 1$

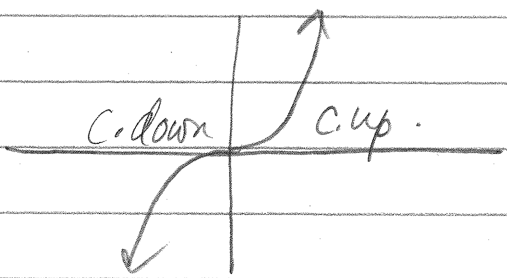
$\boxed{\text{FDT}}$

$f'(-1) = 3 \quad f' = 0 \quad f'(1) = 3$

-1 | $c = 0$ neither max nor min | 1

$f \uparrow$ | | $f \uparrow$

A look at the graph shows f has an inflection pt at $x = 0$.



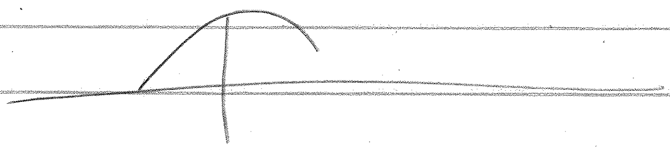
$\boxed{\text{SDT}}$ $f''(x) = 6x$. Testing $x = -1$ & 1 again

$f''(-1) = -6 < 0 \rightarrow f$ c.down on $(-\infty, 0)$

$f''(1) = 6 > 0 \rightarrow f$ c. up on $(0, \infty)$

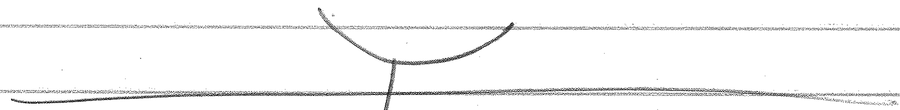
We have $f''(x) = 2x - 1$,

$f''(-3) = -6 - 1 = -7 < 0 \to$ c. down



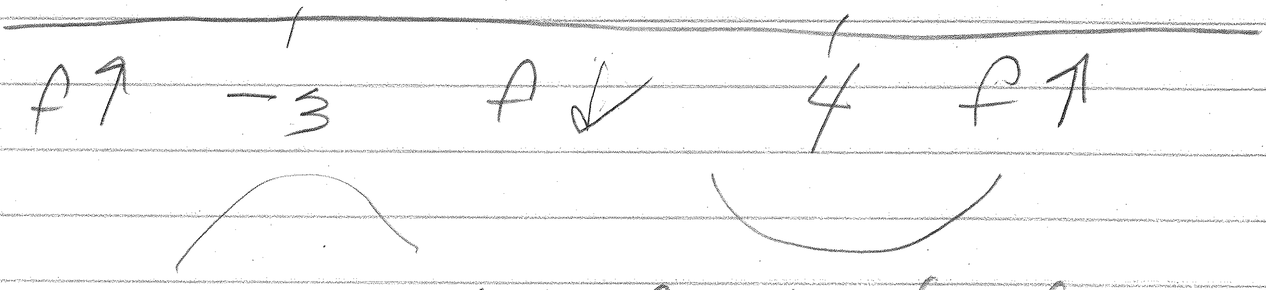
$c = -3$ local max

$f''(4) = 8 - 1 = 7 > 0 \to$ c. up.



$c = 4$ local min.

At some pt, this process becomes automatic. You look at the number line with the c values + do the $f'(c)$, $f''(c)$ & draw conclusions



d) Find the values of the local extremes

Evaluate in the orig. fcn $f(x)$.

$$f(-3) = \frac{2}{3}(-3)^3 - (-3)^2 - 24(-3) - 10$$

$$f(4) = \frac{2}{3}(4)^3 - (4)^2 - 24(4) - 10$$

Horrible numbers!