Sections 10, 11 - A summary with notes

2/23 - 2/25 Returned Test 1 - special attention on limit definition of continuity. See website notes of "Test 1 solution notes".

Sec. 10 takes us to formulas for instantaneous rate of change (i.e., the derivative), some of which we have proved in some way or another by applying the definition of derivative to a fcn. $f(x)$:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}.$$

The primary purpose of the derivative in Mgmt Calculus is to determine "marginal" functions for cost, revenue, and profit. A marginal function tells us the cost, revenue, or profit gained (or lost) by producing or selling "one more item". When we had only linear functions, marginal cost and revenue were simply slope of the fcn. Now we have nonlinear fcns, so these marginal amounts vary with x.

It would be inconvenient to apply the limit def each time! So we memorize some derivative formulas.

First, a word on notation:

$f'(x)$ is often written as y' or dy/dx

$$f'(x) \equiv y' \equiv dy/dx$$

We say "f prime", "y prime" and just "dy/dx" for the ratio of the differentials dy, dx.

Rules of differentiation:

1. $f(x) = c$ (constant) $\rightarrow f'(x) = 0$
2. $f(x) = cx \longrightarrow f'(x) = c$ (the coeff)
3. $f(x) = x^n \longrightarrow f'(x) = nx^{n-1}$, $n \in \mathbb{R}$
4. $(f(x) + g(x))' = f'(x) + g'(x)$

   This rule justifies taking derivatives term-by-term in a multi-term fcn.
5. $(c\,f(x))' = c\,f'(x)$ The coefficient goes along for the ride.

6. So far, we are dealing with single functions or sums of single functions. When we go

to products and quotients of fcns, it's helpful to consider the fcns within these as $u(x)$ and $v(x)$. Call them u and v for short.

6. $f(x) = u \cdot v \longrightarrow f'(x) = u'v + v'u$

7. $f(x) = \frac{u}{v} \longrightarrow f'(x) = \frac{u'v - v'u}{v^2}$

Special case of the quotient rule (7):

$$f(x) = \frac{1}{v} \longrightarrow f'(x) = \frac{-v'}{v^2}$$

You don't have to memorize this. Since $u(x) = 1$, the quotient rule gives this:

$$f'(x) = \frac{0 \cdot v - v' \cdot 1}{v^2} = \frac{-v'}{v^2}$$

Finally, the derivative of the exponential fcn is itself; a proof of this rule is on p. 166.

8. $f(x) = e^x \longrightarrow f'(x) = e^x$

This is only true for base e. There's another rule for other bases:

9. $f(x) = a^x \longrightarrow f'(x) = a^x \ln a$

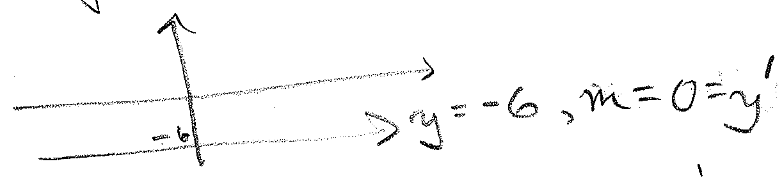
Lastly, the chain rule is used when a <u>fcn.</u> is raised to a power n:

10. $y = [f(x)]^n \longrightarrow y' = n[f(x)]^{n-1} f'(x)$

You can see why notation y' and dy/dx helps. I didn't want to write $([f(x)]^n)'$ because it's not the best form.

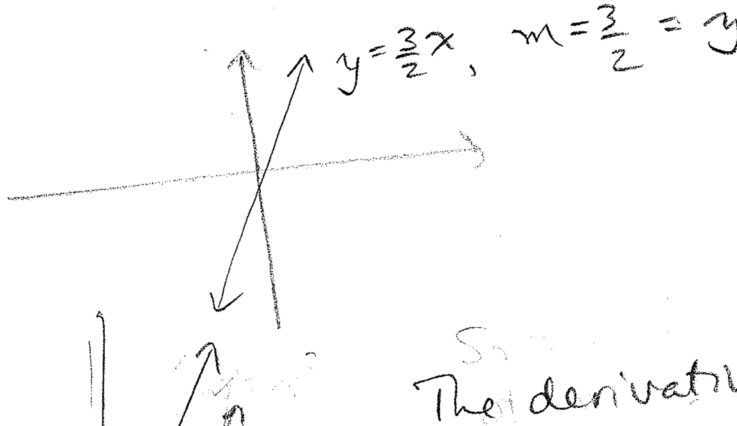
<u>Examples of each rule with graphical interpretation</u>

1. $f(x) = -6$
   $f'(x) = 0$

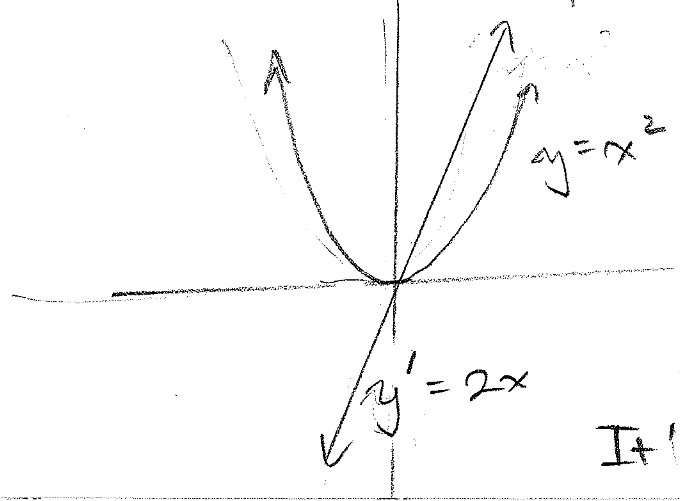


2. $f(x) = \frac{3}{2}x$

   $f'(x) = \frac{3}{2}$



3. $f(x) = x^2$
   $f'(x) = 2x$



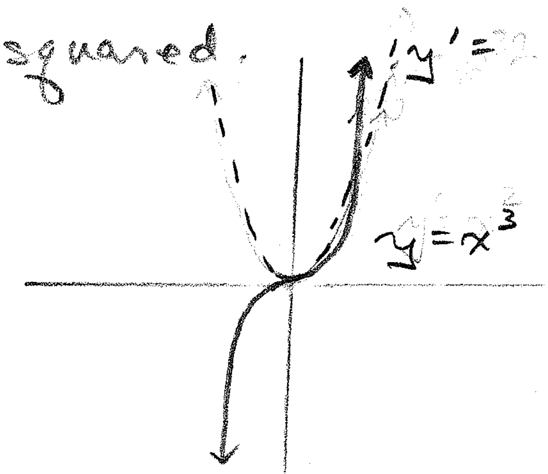
The derivative is negative on $(-\infty, 0)$ and positive on $(0, \infty)$. It's zero at $x = 0$.

4. $f(x) = 16x^3 - 3x^2 + 7x - 11$

$f'(x) = 48x^2 - 6x + 7$

Rather than draw the 2 graphs of these, just note the degree of f' is one less than f's. For a simpler cubic $y = x^3$, $y' = 3x^2$, and you see how the derivative is always positive since x is squared.

I drew the derivative as a dotted line.



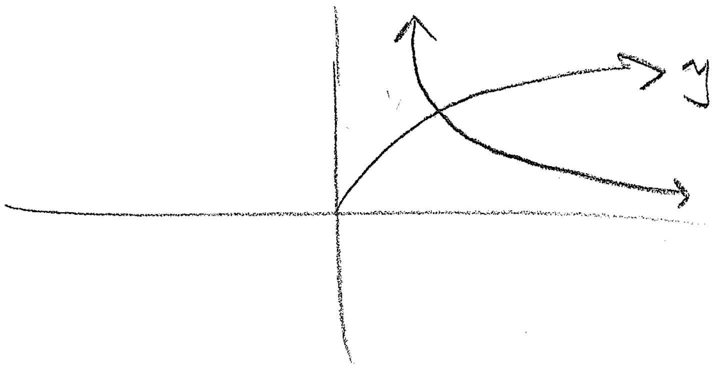
5. $y = 18(x^3 + 2x - 1)$

$y' = 18(3x^2 + 2)$

If you expanded y and then applied rule 4, you'd get the same derivative.

3. again) $f(x) = 2\sqrt{x}$ Write as $f(x) = 2x^{1/2}$

$f'(x) = \frac{1}{2} \cdot 2 \cdot x^{-1/2} = x^{-1/2} = \frac{1}{\sqrt{x}}$



$y' = \frac{1}{\sqrt{x}}$ (very similar to $y = \frac{1}{x}$ but domain is $(0, \infty)$ only

6. $f(x) = \sqrt{x}\,(x^2 - 3)$

$$f'(x) = \frac{d}{dx}\sqrt{x} \cdot (x^2-3) + \sqrt{x} \cdot \frac{d}{dx}(x^2-3)$$

$$= \frac{1}{2}x^{-1/2}(x^2-3) + x^{1/2}\,(2x)$$

Simplified: $f'(x) = \dfrac{x^2-3}{2\sqrt{x}} + 2x\sqrt{x}$

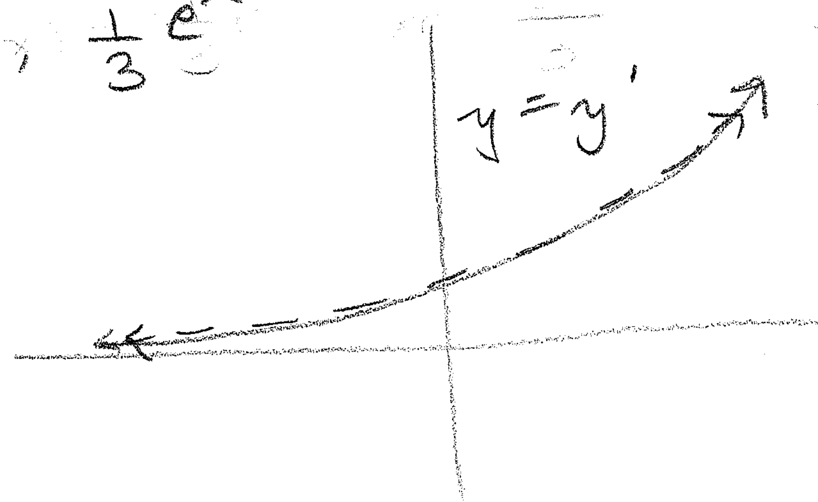
7. $f(x) = \dfrac{x^2 - 1}{x+3}$ $\begin{array}{l} \longrightarrow u(x) \\ \longrightarrow v(x) \end{array}$

$$f'(x) = 2x\,(x+3) - (x^2-1)\cdot 1$$

$$= 2x^2 + 6x - x^2 + 1 = x^2 + 6x + 1$$

8. $f(x) = \dfrac{e^x}{3}$ that is, $\frac{1}{3}e^x$

$$f'(x) = \frac{1}{3}e^x$$



9. $f(x) = 3^x \qquad f'(x) = 3^x \ln 3$

10. $y = \underbrace{(x^2 - 3x + 6)}_{f(x)}^{2} \leftarrow n = 2$

$y' = 2(x^2 - 3x - 6) \cdot (2x - 3)$

Don't expand this

Look again at the chain rule. The fcn. that's raised to a power n might itself be a product or quotient or whatever. Just keep track of the fcns. involved. Also, to apply the chain rule to e^x type fcns, think:

$y = e^{u(x)}$ where the power is a fcn other than x alone.

$$y = e^{u(x)} \rightarrow y' = e^{u(x)} \cdot \frac{du}{dx}$$

For example:

$$y = e^{3x-2} \rightarrow y' = e^{3x-2} \cdot \frac{d}{dx}(3x-2)$$

$$= e^{3x-2} \cdot 3 = 3e^{3x-2}$$