10.2 HW solns - continued

11. a) $\log_6 12 + \log_6 3 - \ln 1$

$= \log_6 (12)(3) - \ln 1$

$= \log_6 36 - \ln 1 \quad = 2 - 0 = \boxed{2}$

b) $\frac{2}{3}\log_4 8 + \frac{1}{2}\log_4 9 - \log_4 6$

$= \log_4 8^{2/3} + \log_4 9^{1/2} - \log_4 6$

$= \log_4 2^2 + \log_4 3 - \log_4 6$

$= \log_4 4 + (\log_4 3 - \log_4 6)$

$= 1 + \log_4 3/6 = 1 + \log_4 1/2$

Now, $\log_4 1/2 = ?$ i.e., $4^? = 1/2$

Since $4^{1/2} = 2$, then $4^{-1/2} = \frac{1}{4^{1/2}} = \frac{1}{2}$

So $? = -1/2$.

Hence, $1 + \log_4 1/2 = 1 + -\frac{1}{2} = \boxed{\frac{1}{2}}$

#12a) $\log_a \frac{1}{n} = \log_a(n^{-1}) = -1 \log_a n = -\log_a n$

b) $\log_a \frac{1}{n} = \log_a(n^{-1}) = -1 \log_a n = -\log_a n$

Wait, these are identical. Oh, use different properties? OK:

$$\log_a \frac{1}{n} = \log_a 1 - \log_a n = \cancel{0} - \log_a n \checkmark$$

#16) $\log_a b = \dfrac{\log_b b}{\log_b a} = \dfrac{1}{\log_b a}$

#15) $\log_7 9 = \log_5 9 / \log_5 7$

$= \log 9 / \log 7$

$= \ln 9 / \ln 7$

Caution! $\log 9 / \log 7$ (for ex.) $\neq \log 9 - \log 7 = \log\left(\frac{9}{7}\right)$

#19. $\ln 2 \approx 0.69$, $\ln 3 \approx 1.10$, $\ln 5 \approx 1.61$ $\longrightarrow$ Use these to approximate the following:

a) $\ln 15 = \ln (5)(3) = \ln 5 + \ln 3 \approx 1.61 + 1.10 \approx \boxed{2.71}$

b) $\ln 1.5 = \ln (3/2) = \ln 3 - \ln 2 \approx 1.10 - 0.69 = \boxed{0.31}$

#19 c) $\ln \sqrt[3]{2} = \ln 2^{1/3} = \frac{1}{3}\ln 2 \approx \frac{1}{3}(0.69) \approx \boxed{0.23}$

d) $\ln 0.9 = \ln \frac{9}{10}$

$= \ln\left(\frac{3^2}{(2)(5)}\right)$

$= \ln 3^2 - \ln(2)(5)$

$= 2\ln 3 - (\ln 2 + \ln 5)$

$\approx 2(1.10) - (0.69 + 1.61)$

$\approx 2.2 - 2.30 = \boxed{-0.1}$

#18 Use $a^x = b^{x \log_b a}$ to change these to base e

a) $3^x = e^{x \ln 3}$

b) $6^x = e^{x \ln 6}$

c) $10^x = e^{x \ln 10}$

#20 Approximate; given $\log 3 \approx 0.477$

a) $\log 30 = \log(3)(10) = \log 3 + \log 10$

$\approx 0.477 + 1 = \boxed{1.477}$

b) $\log 3000 = \log(3)(1000) = \log 3 + \log 1000$

$= \log 3 + 3 \approx \boxed{3.477}$

c) $\log_3 10$

change of base:

AACK!

$\log_3 10 = \frac{\log 10}{\log 3} \approx \boxed{\frac{1}{.477}}$

#21. (a) $\log_9 21 = \frac{\log 21}{\log 9} = \frac{\ln 21}{\ln 9}$

$$\log_9 21 = \frac{\log_3 21}{\log_3 9} = \frac{\log_3 21}{2}$$

$$= \frac{1}{2}\log_3 21 = \log_3 (21)^{1/2} = \log_3 \sqrt{21}$$

#22) Solve these exponential eqns for x

#23) Solve these logarithmic eqns for x

See final HW for Ch. 10!