Definition 3.4.1.

Suppose f is a one-to-one function with domain X and range Y. We define the <u>inverse function of f</u>, *denoted "f^{-1}" to be the function with domain Y and range X whose relationship with f is:*

$$f^{-1}[f(x)] = x \quad AND \quad f[f^{-1}(y)] = y$$

Important Idea 3.4.1.

The notation $f^{-1}(x)$ means the function that is the inverse *of function f. It does not mean the function $\frac{1}{f(x)}$.*

Comprehension Check 3.4.

For an inverse function to exist for f it is required that f be a one-to-one function. Explain why.

If f is a set of points (x, y), then f^{-1} is the set of points (y, x). We use this fact to conclude that the graphs of f and f^{-1} are reflections of each other over the line $y = x$. The line $y = x$ is the line that goes through the origin at a 45° angle and bisects the first and third quadrants. Let's illustrate this with a finite example. Have your pencil ready.

Suppose we have $f(x) = \{(-2,-1),(-1,1),(0,3),(1,0),(2,2),(3,-4)\}$. Since f is a one-to-one function, it has an inverse function f^{-1}. We know that for every point (x, y) belonging to f there is a point (y, x) which belongs to f^{-1}. So, $f^{-1}(x) = \{(-1,-2),(1,-1),(3,0),(0,1),(2,2),(-4,3)\}$. Draw a set of coordinate axes and scale it fairly carefully in integer units, including at least $[-4, 4]$ in both the vertical and horizontal directions. Draw in the line $y = x$. Plot the points that comprise the function f. In another color (or just write lighter with your pencil) plot the points that comprise the function f^{-1}. To help you see that the two collections of points are indeed mirror images about the mirror $y = x$ you could draw line segments between each pair $(-2,-1)$ *and* $(-1,-2)$, $(-1,1)$ *and* $(1,-1)$, etc. These line segments should be perpendicular to the line $y = x$ and should be bisected by the line $y = x$. Notice that there is one point that is actually *on* the line $y = x$. Not surprisingly, this is the point $(2, 2)$ whose y-value is equal to its x-value.

While studying this example we need to notice the relationship between the domains and ranges of f and f^{-1}. The domain of f is $\{-2,-1,0,1,2,3\}$. The range of f is $\{-4,-1,0,1,2,3\}$. The domain of f^{-1} is $\{-4,-1,0,1,2,3\}$ and the range is $\{-2,-1,0,1,2,3\}$.

We repeat for emphasis the characteristics of inverse functions:

Important Idea 3.4.2.

If f and f^{-1} are inverse functions, then:

1. *Both f and f^{-1} are one-to-one functions.*
2. $f[f^{-1}(x)] = x \quad AND \quad f^{-1}[f(x)] = x$
3. $D_f = R_{f^{-1}} \quad AND \quad D_{f^{-1}} = R_f$
4. *If $f = \{(x, y)\}$, then $f^{-1} = \{(y, x)\}$*
5. *The graphs of f and f^{-1} are reflections of each other over the line $y = x$.*

Example 3.4.1.

Show by the definition that $f(x) = \frac{1}{2}x - 3$ and $f^{-1}(x) = 2x + 6$ are inverses of each other.

We need to show that $f[f^{-1}(x)] = x \quad AND \quad f^{-1}[f(x)] = x$.

$f[f^{-1}(x)] = f(2x+6) = \frac{1}{2}(2x+6) - 3 = x + 3 - 3 = x \quad \checkmark$

$f^{-1}[f(x)] = f^{-1}(\frac{1}{2}x - 3) = 2(\frac{1}{2}x - 3) + 6 = x - 6 + 6 = x \quad \checkmark$

On a set of carefully drawn coordinate axes draw a dotted line to represent the line $y = x$. Now graph the functions f and f^{-1} from Example 3.4.1. These functions are both lines whose graphs we will study in the Polynomials chapter. You can graph each one now by plotting its x-intercept and y-intercept and drawing a straight line through its itercepts. Did your graphs come out to be refections of each other about the line $y = x$? (They should). Where do the two lines f and f^{-1} intersect? If your graphs were drawn carefully, the intersection point is on the line $y = x$. Algebraically figure out what the exact coordinates of that point must be.

Example 3.4.2.
Show by the definition that $f(x) = \dfrac{3}{x-1}$ *and* $f^{-1}(x) = \dfrac{3}{x} + 1$ *are inverses of each other.*

We need to show that $f[f^{-1}(x)] = x$ *AND* $f^{-1}[f(x)] = x$.

$$f[f^{-1}(x)] = f\left(\frac{3}{x} + 1\right) = \frac{3}{(\frac{3}{x} + 1) - 1} = \frac{3}{\frac{3}{x}} = x \quad \checkmark$$

$$f^{-1}[f(x)] = f^{-1}\left(\frac{3}{x-1}\right) = \frac{3}{\frac{3}{x-1}} + 1 = (x-1) + 1 = x \quad \checkmark$$

In the previous two examples we were given pairs of functions that were inverses and had to verify their relationship. If we are given a one-to-one function, how can we find its inverse? We use the definition.

Example 3.4.3.
Find $f^{-1}(x)$*, the inverse function for* $f(x) = 2x^3 + 1$.

We know that if f^{-1} *is the inverse function for* f *then* $f[f^{-1}(x)] = x$. *We will start with that equation and then solve for* $f^{-1}(x)$.

$$\begin{aligned} f[f^{-1}(x)] &= x \\ 2[f^{-1}(x)]^3 + 1 &= x \\ 2[f^{-1}(x)]^3 &= x - 1 \\ [f^{-1}(x)]^3 &= \frac{x-1}{2} \\ f^{-1}(x) &= \sqrt[3]{\frac{x-1}{2}} \end{aligned}$$

We have a candidate for f^{-1}. *We got it by assuming that* $f[f^{-1}(x)] = x$. *We need to verify that our candidate is indeed the inverse by checking the reverse equation:* $f^{-1}[f(x)] = x$.

$$f^{-1}[f(x)] = f^{-1}(2x^3 + 1) = \sqrt[3]{\frac{(2x^3+1) - 1}{2}} = \sqrt[3]{\frac{2x^3}{2}} = \sqrt[3]{x^3} = x \quad \checkmark$$

Example 3.4.4.

Find the domain and range of the function $f(x) = \dfrac{x+2}{x-3}$

We cannot have a zero in the denominator of the function so the domain for f is $\{x \in \Re : x \neq 3\}$.

We know that the range of f is the domain of its inverse, so we find $f^{-1}(x)$.

$$\begin{aligned}
f[f^{-1}(x)] &= x \\
\frac{f^{-1}(x)+2}{f^{-1}(x)-3} &= x \\
f^{-1}(x)+2 &= [f^{-1}(x)-3]x \\
f^{-1}(x)+2 &= [f^{-1}(x)]x - 3x \\
f^{-1}(x) - [f^{-1}(x)]x &= -3x-2 \\
[f^{-1}(x)](1-x) &= -3x-2 \\
f^{-1}(x) &= \frac{-3x-2}{1-x}
\end{aligned}$$

We have a candidate for f^{-1}. *We need to check it in* $f^{-1}[f(x)] = x$.

$$f^{-1}[f(x)] = \frac{-3\left(\frac{x+2}{x-3}\right)-2}{1-\left(\frac{x+2}{x-3}\right)} = \frac{-3(x+2)-2(x-3)}{(x-3)-(x+2)} = \frac{-3x-6-2x+6}{x-3-x-2} = \frac{-5x}{-5} = x \quad \checkmark$$

We have verified that $f^{-1}(x) = \dfrac{-3x-2}{1-x}$. *The domain for* $f^{-1}(x)$ *is* $\{x \in \Re : x \neq 1\}$ *so this set is the range for f.*

Many students learn to find the inverse of a function by taking the original function, writing it in terms of x and y, then changing the x's to y's and the y's to x's, and then solving for y. This is fine for a mechanical method and the notation can be simpler. This process really does the same thing as the first step of the process above. To verify that the found function is the inverse, though, it still needs to be checked in the second composition. One draw-back to the "just change the x's and y's" method is that it is difficult to keep track of which function is which. So, it is important that at the end of the work that the function f^{-1} is specifically identified, and not just left with the ambiguous designation of "y."

It is important when verifying that two functions are inverses that we check BOTH directions of the composition of their functions. It is possible that a composition works one way but not the other. These functions, then would not be inverses.

Consider $f(x) = x^2$ and $g(x) = \sqrt{x}$. $f[g(x)] = f(\sqrt{x}) = (\sqrt{x})^2 = x$. It is looking like f and g might be inverses of each other. But look what happens when we try to go the other way. $g[f(x)] = g(x^2) = \sqrt{(x^2)} = |x|$. f and g are not inverses of each other.

In Exercise 3.5 of Section 3.3 we saw that $f(x) = x^2$ is not one-to-one. So, it cannot have an inverse. The domain for $f(x) = x^2$ is $\Re$, but the range for $g(x) = \sqrt{x}$ is not $\Re$. This is precisely the problem we ran into when we tried to show that f and g were inverses.

The function "rules" for inverse functions are such that one reverses the effect of the other. This is why their compositions always return the original input: $f[f^{-1}(x)] = x$ and $f^{-1}[f(x)] = x$. We think of the squaring operation and the taking of a square root as each operation reversing the other. We would like to think of them as inverses, but strictly they are not. There is the domain problem. However, we can recognize the domain problem and pose another function to be the inverse of $g(x) = \sqrt{x}$. We suggest that $g^{-1}(x) = x^2$ for $x \geq 0$. If we restrict the domain of the squaring function, then we do have an inverse function for the square root function. The problem we had before with $g[f(x)] = g(x^2) = \sqrt{(x^2)} = |x|$ goes away if we restrict $x \geq 0$. We will look at these particular functions in more detail in the Family of Simple Functions chapter.

3.5 Exercises

Problems for Section 3.1

Problem 1. Which of the following are functions? If not a function, tell why not. Assume x would represent the independent variable.

(a) $y = x^2$ (b) $x^2 = y - 2x$ (c) $\{(2,-1),(4,3),(1,2),(0,3)\}$

(d) $y = \begin{cases} -x & if \quad x < 0 \\ 2 & if \quad x > 0 \end{cases}$ (e) $y^2 = x$ (f) $\{(4,2),(-3,1),(2,4),(-3,-2)\}$

Problem 2. Given f where $D_f = \{-1,0,1,3,7\}$ and $f(x) = x^2 + 3$, find R_f, the range of f.

Problem 3. For each of the following functions, find the domain and range. Then evaluate the function at $-1, 0, 1$ and 2 if in the domain.

(a) $f(x) = \dfrac{1}{x-2}$ (b) $f(x) = \{(2,1),(4,3),(1,2),(0,-3)\}$

(c) $f(x) = \sqrt{2-x} + 1$ (d) $A(r) = \pi r^2$ (area of a circle as a function of its radius)

Problem 4. For each of the following functions, find the domain:

(a) $f(x) = x^2 - x + 2$ (b) $y = \dfrac{x-5}{3x^2+12x}$ (c) $g(x) = \dfrac{1}{x} + \dfrac{4}{x+3}$ (d) $f(x) = \sqrt{\dfrac{x^2}{\cdot}}$

(e) $h(x) = \sqrt[4]{x+5}$ (f) $s(x) = \sqrt[3]{3x+7}$ (g) $y = \dfrac{x+2}{x^2-4}$

Problem 5. Given $f(x) = (3+x)^2$, show that $f(2) + f(3) \neq f(5)$.

Problem 6. For each of the following functions find the domain and evaluate the function for $x = -2, -\frac{1}{2}, 0, 1$ and $\frac{9}{4}$ if in the domain. Also, for functions g and h, graph the function and give its range.

(a) $f(x) = \begin{cases} \sqrt{1-x^2} & if \quad -1 \leq x \leq 1 \\ \frac{1}{x} & if \quad x > 1 \end{cases}$ (b) $g(x) = \begin{cases} -3 & if \quad x < 0 \\ x & if \quad x > 0 \end{cases}$

(c) $h(x) = \begin{cases} -1 & if \quad x \text{ is an integer} \\ 1 & if \quad x \text{ is not an integer} \end{cases}$

Problem 7. The cost of renting a car for a week is \$250 plus 25 cents per mile driven. Write a function that describes the cost of week's rental as a function of the number of miles driven. Identify your independedent variable, your dependent variable and the function's domain and range.

Problem 8. Write an equation that expresses the area of a square in terms of its perimeter.

Problem 9. The length of a rectangle is three inches more than its width. Write an equation that expresses the area of the rectangle as a function of its width. Then write an equation that expresses the area of the rectangle as a function of its length.

Problems for Section 3.2

Problem 1. Given $f(x) = \frac{1}{x}$ and $g(x) = x$. Does $(f \circ f) = g$? Justify your answer.

Problem 2. Show that the domains for $f(x) = \sqrt{\dfrac{x}{x-2}}$ and $g(x) = \dfrac{\sqrt{x}}{\sqrt{x-2}}$ are not the same. This means that $f \neq g$.

Problem 3. Given functions $f(x) = \dfrac{3}{x+1}$ and $g(x) = \dfrac{x+2}{x-1}$, find:

(a) $(f+g)$ and the domain of $(f+g)$

(b) $(f-g)$ and the domain of $(f-g)$

(c) $(f \cdot g)$ and the domain of $(f \cdot g)$

(d) $\dfrac{f}{g}$ and the domain of $\dfrac{f}{g}$

(e) $(f+g)(2)$ and $(f \cdot g)(-2)$

Problem 4. Given functions f and g below, find $(f+g)(0)$, $(f+g)(5)$ and function $(f+g)(x)$

$$f(x) = \begin{cases} x+3 & if \quad x \leq 0 \\ 2x-5 & if \quad x > 0 \end{cases} \qquad g(x) = \begin{cases} 3x-1 & if \quad x < -1 \\ 4-x & if \quad x \geq -1 \end{cases}$$

Problem 5. Given $f(x) = 1 - 2x^2$ and $g(x) = x + 1$, find $(f \circ g)$, $(g \circ f)$, $(f \circ f)$, $(g \circ g)$.

Problem 6. For each of the following pairs f and g, find $(f \circ g)$ and $(g \circ f)$ and their domains.

(a) $f(x) = \sqrt{x} \quad g(x) = x^2$

(b) $f(x) = x + 2 \quad g(x) = \dfrac{1}{x-3}$

Problem 7. For functions f and g below, find $(f \circ g)(4)$, $(f \circ g)(-4)$ and $(f \circ g)(x)$.

$$f(x) = \begin{cases} 2x & if \quad x < 0 \\ x^2 & if \quad x > 3 \end{cases} \qquad g(x) = 1 - x$$

Problem 8. Find $(f \circ g)$ if $f = \{(-1,2), (0,7), (2,5)\}$ and $g = \{(-1,1), (3,0), (4,-1)\}$.

Problem 9. Evaluate the Difference Quotient (see page 75) for the following functions:

(a) $f(x) = \dfrac{1}{x}$

(b) $f(x) = x^2 + x$

Problem 10. For each of the following functions h, find functions f and g such that $h = f \circ g$. Do not use $f(x) = x$ or $g(x) = x$.

(a) $h(x) = \sqrt{x-4}$ (b) $h(x) = \dfrac{2}{x^5 - 2x^3 + 1}$ (c) $h(x) = \sqrt{\sqrt{x}+1}$

Problems for Section 3.3

Problem 1. For each of the following functions, find the y-intercept and x-intercept(s).

(a) $f(x) = (2x-1)(5+x)$ (b) $f(x) = \dfrac{2x-3}{5+x}$ (c) $f(x) = 2x^3 - 3x^2 + 2x - 3$
(d) $f(x) = \sqrt{x+9} - 5$ (e) $f(x) = \dfrac{-1}{x} + \dfrac{x}{6-x}$

Problem 2. Determine whether the following functions are even, odd, or neither.

(a) $f(x) = 3x^4 - 2x^2 - 5$ (b) $f(x) = x^3 + 5$ (c) $f(x) = \dfrac{x^2+1}{x}$

Problem 3. Given that functions f and g are both odd, show that the function $(f \cdot g)$ is even.

Problem 4. Explain why the function $f(x) = x^2$ is not one-to-one.

Problem 5. Suppose we know that the function f contains the points $(-1,3), (6,2), (-2,-3)$.
(a) What other points must also belong to f if f is even?
(b) What other points must also belong to f if f is odd?
(c) Is it possible that either of the functions described in parts (a) or (b) are one-to-one? Just your answer.

Problem 6. Sketch the graph of a function that is:

(a) Odd and one-to-one. (b) Odd and not one-to-one.
(c) Even and one-to-one. (d) Even and not one-to-one.

Problem 7. Use the definition of "one-to-one" to show that the following functions are one-to-

(a) $f(x) = 2x - 7$ (b) $f(x) = \dfrac{x}{x+2}$

Problems for Section 3.4

Problem 1. Which of the following pairs of functions f and g are inverses of each other?

(a) $f(x) = \sqrt[3]{x-1}$ $g(x) = x^3 + 1$ (b) $f(x) = 5 - x$ $g(x) = x - 5$
(c) $f(x) = 3x - 2$ $g(x) = \frac{1}{3}x + 2$ (d) $f(x) = \frac{1}{4}x + 3$ $g(x) = 4x - 12$

Problem 2. For each of the following functions f, find its inverse function f^{-1}.

(a) $f(x) = 4x + 2$ (b) $f(x) = \{(1,2), (-2,3), (-1,-1)\}$ (c) $f(x) = 2x^5 - 3$
(d) $f(x) = \frac{1}{x}$ (e) $f(x) = \dfrac{4+x}{2x-7}$ (f) $f(x) = 3 - x^2$ for x

Problem 3. A function is one-to-one if, and only if, it has an inverse. Show that $f(x) = 2x - 7$ is a one-to-one function by showing that it has an inverse.

Problem 4. If f and g are inverses of each other and f has a y-intercept, does that mean that g must have an x-intercept? Justify your answer.

Problem 5. Given $f(x) = \dfrac{x}{x+2}$, find f^{-1} and state the domain and range for both functions.

Problem 6. Given functions f and g with inverses f^{-1} and g^{-1}, it is true that $(f \circ g)^{-1} = g^{-1} \circ f^{-1}$. Illustrate this using $f(x) = 2x + 3$ and $g(x) = x - 5$ and the following steps:
(a) Find f^{-1} and g^{-1} (b) Find $(f \circ g)$ (c) Find $(f \circ g)^{-1}$ (d) Find $g^{-1} \circ f^{-1}$
(e) Verify that your answers in parts (c) and (d) are the same.

3.6 Answers to Exercises

Answers for Section 3.1 Exercises

Answer to Problem 1.

(a) Yes (b) Yes (c) Yes (d) Yes (e) No (f) No

Answer to Problem 2.

$R_f = \{3, 4, 12, 52\}$

Answer to Problem 3.

(a) $D_f = \{x \in \Re : x \neq 2\}$ $R_f = \{y \in \Re : y \neq 0\}$
$f(-1) = -\frac{1}{3}$ $f(0) = -\frac{1}{2}$ $f(1) = -1$ $f(2)$ is undefined.

(b) $D_f = \{0, 1, 2, 4\}$ $R_f = \{-3, 1, 2, 3\}$
$f(-1)$ is undefined $f(0) = -3$ $f(1) = 2$ $f(2) = 1$

(c) $D_f = \{x \in \Re : x \leq 2\}$ $R_f = \{y \in \Re : y \geq 1\}$
$f(-1) = \sqrt{3} + 1$ $f(0) = \sqrt{2} + 1$ $f(1) = 2$ $f(2) = 1$

(d) $D_A = \{r \in \Re : r \geq 0\}$ $R_A = \{A(r) \in \Re : A(r) \geq 0\}$
$A(-1)$ is undefined. $A(0) = 0$ $A(1) = \pi$ $A(2) = 4\pi$

Answer to Problem 4.

(a) $D_f = \Re$
(b) $D_y = \{x \in \Re : x \neq 0, x \neq -4\}$
(c) $D_g = \{x \in \Re : x \neq 0, x \neq -3\}$
(d) $D_f = \{x \in \Re : x < 0\}$
(e) $D_h = \{x \in \Re : x \geq -5\}$
(f) $D_f = \Re$
(g) $D_y = \{x \in \Re : x \neq 2, x \neq -2\}$

Answer to Problem 5.

$f(2) = 25, \quad f(3) = 36, \quad f(5) = 64. \qquad 25 + 36 \neq 64.$

Answer to Problem 6.

(a) $D_f = \{x \in \Re : x \geq -1\}$
$f(-2)$ undef $\quad f(-\frac{1}{2}) = \frac{\sqrt{3}}{2} \quad f(0) = 1 \quad f(1) = 0 \quad f(\frac{9}{4}) = \frac{4}{9}$

(b) $D_g = \{x \in \Re : x \neq 0\} \quad R_g = \{y \in \Re : y = -3 \text{ or } y > 0\}$
$g(-2) = -3 \quad g(-\frac{1}{2}) = -3 \quad g(0)$ undef $\quad g(1) = 1 \quad g(\frac{9}{4}) = \frac{9}{4}$

(c) $D_h = \Re \quad R_h = \{-1, 1\}$
$h(-2) = -1 \quad h(-\frac{1}{2}) = 1 \quad h(0) = -1 \quad h(1) = -1 \quad h(\frac{9}{4}) = 1$

Answer to Problem 7.

Let C = total cost (dollars) and let m = distance driven (miles). $C(m) = 250 + .25m$.
$D_C = \{m \in \Re : m \geq 0\}$. $R_C = \{C : C \geq 250\}$.

Answer to Problem 8.

$A(P) = \left(\frac{P}{4}\right)^2$

Answer to Problem 9.

$A(l) = l(l-3) \qquad A(w) = (3+w)w$

Answers for Section 3.2 Exercises

Answer to Problem 1.

No. $\quad D_{f \circ f} = \{x \in \Re : x \neq 0\} \quad D_g = \Re$.

Answer to Problem 2.

Answers will vary. One answer: $-1 \in D_f$, but $-1 \notin D_g$

Answer to Problem 3.

(a) $(f+g) = \dfrac{3}{x+1} + \dfrac{x+2}{x-1} \quad D_{f+g} = \{x \in \Re : x \neq \pm 1\}$

(b) $(f-g) = \dfrac{3}{x+1} - \dfrac{x+2}{x-1} \quad D_{f-g} = \{x \in \Re : x \neq \pm 1\}$

(c) $(f \cdot g) = \left(\dfrac{3}{x+1}\right)\left(\dfrac{x+2}{x-1}\right) \quad D_{f \cdot g} = \{x \in \Re : x \neq \pm 1\}$

(d) $\left(\dfrac{f}{g}\right) = \dfrac{\frac{3}{x+1}}{\frac{x+2}{x-1}} \quad D_{\frac{f}{g}} = \{x \in \Re : x \neq \pm 1, \ x \neq -2\}$

(e) $(f+g)(2)=5 \quad (f\cdot g)(-2)=0$

Answer to Problem 4.

$$(f+g)(x)=\begin{cases} 4x+2 & if \quad x<-1 \\ 7 & if \quad -1\le x\le 0 \\ x-1 & if \quad x>0 \end{cases} \qquad (f+g)(0)=7 \qquad (f+g)(5)=4$$

Answer to Problem 5.

$$(f\circ g)=-2x^2-4x-1 \quad (g\circ f)=-2x^2+2 \quad (f\circ f)=-8x^4+8x^2-1 \quad (g\circ g)=x+2$$

Answer to Problem 6.

(a) $(f\circ g)(x)=\sqrt{x^2}$ $\quad D_{fog}=\Re$ $\quad (g\circ f)(x)=(\sqrt{x})^2$ $\quad D_{gof}=\{x\in\Re: x\ge 0\}$

(b) $(f\circ g)(x)=\frac{1}{x-3}+2$ $\quad D_{fog}=\{x\in\Re: x\ne 3\}$ $\quad (g\circ f)(x)=\frac{1}{(x+2)-3}$ $\quad D_{gof}=\{x\in\Re: x\ne 1\}$

Answer to Problem 7.

$$(f\circ g)(x)=\begin{cases} (1-x)^2 & if \quad x<-2 \\ 2(1-x) & if \quad x>1 \end{cases} \qquad (f\circ g)(4)=-6 \qquad (f\circ g)(-4)=25$$

Answer to Problem 8.

$(f\circ g)=\{(3,7),(4,2)\}$

Answer to Problem 9.

(a) $-\dfrac{1}{x(x+h)}$ $\quad$ (b) $2x+1+h$

Answer to Problem 10.

Answers will vary.

Answers for Section 3.3 Exercises

Answer to Problem 1.

(a) y-int$=-5$ $\quad x$-ints$=\frac{1}{2},-5$

(b) y-int$=-\frac{3}{5}$ $\quad x$-int$=\frac{3}{2}$

(c) y-int$=-3$ $\quad x$-int$=\frac{3}{2}$

(d) y-int$=-2$ $\quad x$-int$=16$

(e) y-int$=$ *none* $\quad x$-ints$=-3,2$

Answer to Problem 2.

(a) even $\qquad$ (b) neither $\qquad$ (c) odd

Answer to Problem 3.

Proof. Solution not given.

Answer to Problem 4.
Explanations will vary.

Answer to Problem 5.

(a) $(1,3),(-6,2),(2,-3)$ (b) $(1,-3),(-6,-2),(2,3)$ (c) No. Neither is one-to-one.

Answer to Problem 6.
Answers will vary.

Answer to Problem 7.
Proof. Solution not given.

Answers for Section 3.4 Exercises

Answer to Problem 1.

(a) Yes (b) No (c) No (d) Yes

Answer to Problem 2.

(a) $f^{-1}(x) = \dfrac{x-2}{4}$ (b) $f^{-1}(x) = \{(2,1),(3,-2),(-1,-1)\}$ (c) $f^{-1}(x) = \sqrt[5]{\dfrac{x+3}{2}}$

(d) $f^{-1}(x) = \dfrac{1}{x}$ (e) $f^{-1}(x) = \dfrac{4+7x}{2x-1}$ (f) $f^{-1}(x) = \sqrt{3-x}$

Answer to Problem 3.

$$f^{-1}(x) = \frac{x+7}{2}$$

Answer to Problem 4.

Yes

Answer to Problem 5.

$f^{-1}(x) = \dfrac{2x}{1-x}$ $D_f = R_{f^{-1}} = \{x \in \Re : x \neq -2\}$ $D_{f^{-1}} = R_f = \{x \in \Re : x \neq 1\}$

Answer to Problem 6.

(a) $f^{-1} = \dfrac{x-3}{2}$ and $g^{-1} = x+5$ (b) $(f \circ g) = 2x - 7$

(c) $(f \circ g)^{-1} = \dfrac{x+7}{2}$ (d) $(g^{-1} \circ f^{-1}) = \dfrac{x+7}{2}$