Math 220-04 Take-home quiz
Due 13 November 2015

Key

For the function $f(x,y) = 6y^2 - 2y^3 + 3x^2 + 6xy$, write out all first and second order partial derivatives

$f_x = 6x + 6y$ $f_y = 12y - 6y^2 + 6x$

$f_{xx} = 6$ $f_{yy} = 12 - 12y$

$f_{xy} = 6$

Find the critical values (x_o, y_o) of the function, that is, those points where $f_x = 0$ and $f_y = 0$.

$f_x = 6x + 6y = 0$
$6(x+y) = 0$
$x + y = 0$
① $x = -y$
(so, $(0,0)$ is one soln)

$f_y = 12y - 6y^2 + 6x = 0$
chk $(0,0)$: $f_y = 0 - 0 + 0 = 0$
$f_y = 12y - 6y^2 + 6x$
$= 6(2y - y^2 + x) = 0$
$= 6(2y - y^2 - y) = 0$
$6(y - y^2) = 0$
$6y(1 - y) = 0$
② $y = 1$
So $x = -1$ from ①

* Solve the equation of the *discriminant* for *each* of the critical points you found:

$D = f_{xx} f_{yy} - f_{xy}^2$
$= 6(12 - 12y) - 6^2$
$D(0,0) = 72 - 36 = 36 > 0$

$f_{xx}(0,0) = 6 > 0$
$\therefore (0,0)$ local min.

Determine if these values constitute local maxima, minima, or saddle points using the criteria:

If $D < 0$ then (x_o, y_o) is a saddle point.

If $D > 0$ then (x_o, y_o) is $\begin{cases} \text{maximum if } f_{xx} < 0 \text{ or } f_{yy} < 0 \\ \text{minimum if } f_{xx} > 0 \text{ or } f_{yy} > 0 \end{cases}$

Finally, give the value of the function at the various extremes and/or saddle points.

$f(0,0) = 0 - 0 + 0 + 0 = 0$
$f(-1,1) = 6 - 2 + 3 - 6 = 1$

$\boxed{f(0,0) = 0 \quad f(-1,1) = 1}$

* Rest: $D(-1,1) = 6(12 - 12(1)) - 36 = -36 < 0$
$\therefore (-1,1)$ saddle pt.

Finally: $f(0,0) = -1, \quad f(\sqrt{2},-2) = 3, \quad f(-\sqrt{2},-2) = 3$

Do the same for the function:

$$f(x,y) = 4x^2 + y^2 + 2x^2y - 1$$

$f_x = 8x + 4xy \qquad f_y = 2y + 2x^2$

$f_{xx} = 8 + 4y \qquad f_{yy} = 2$

$f_{xy} = 4x \quad (f_{yx} = 4x \text{ also})$

$f_x = 8x + 4xy = 0 \qquad f_y = 2y + 2x^2 = 0$

$4x(2+y) = 0 \qquad$ ② $2(y+x^2) = 0$

$4x = 0$ or $2+y=0$ $\qquad$ ① into ② $\to$

① $\boxed{x=0}$ or $\boxed{y=-2}$ $\qquad$ f_y: $2(y+0^2) = 0$

$2y = 0$

$\boxed{y=0}$

$(0,0)$

crit pt

① into
from ② also, $2(y+x^2) = 2(-2+x^2) = 0$

$x^2 = 2, \quad x = \pm\sqrt{2}$

Do $(\sqrt{2},-2)$, $(\sqrt{2},2)$, $(0,0)$ all check into

$f_x = 0$ & $f_y = 0$? yes (do this)

Chk $f_x = 8x + 4xy, \quad f_y = 2y + 2x^2$ for $(0,0), (\pm\sqrt{2},-2)$

Finally, $D(x,y) = (8+4y)(2) - (4x)^2 = 16 + 8y - 16x^2$

$D(0,0) = 16 > 0, \quad f_{xx} = 8 + 4(0) = 8 > 0 \therefore (0,0)$ local min

$D(\sqrt{2},-2) = (8+4(-2))(2) - (4\sqrt{2})^2 = -32 < 0 \therefore (\sqrt{2},-2)$ saddle pt

$D(-\sqrt{2},-2) = (8+4(-2))2 - (4(-\sqrt{2}))^2 = -32 < 0$

$\therefore (-\sqrt{2},-2)$ saddle pt