Excluding Values from the Domain of $(f \circ g)(x) = f(g(x))$

The following values must be excluded from the input x:

- If x is not in the domain of g, it must not be in the domain of $f \circ g$.
- Any x for which $g(x)$ is not in the domain of f must not be in the domain of $f \circ g$.

EXAMPLE 5 Forming a Composite Function and Finding Its Domain

Given $f(x) = \dfrac{2}{x-1}$ and $g(x) = \dfrac{3}{x}$, find each of the following:

a. $(f \circ g)(x)$ **b.** the domain of $f \circ g$.

Solution

a. Because $(f \circ g)(x)$ means $f(g(x))$, we must replace x in $f(x) = \dfrac{2}{x-1}$ with $g(x)$.

$$(f \circ g)(x) = f(g(x)) = \frac{2}{g(x)-1} = \frac{2}{\frac{3}{x}-1} = \frac{2}{\frac{3}{x}-1} \cdot \frac{x}{x} = \frac{2x}{3-x}$$

$g(x) = \frac{3}{x}$

Simplify the complex fraction by multiplying by $\frac{x}{x}$, or 1.

Thus, $(f \circ g)(x) = \dfrac{2x}{3-x}$.

b. We determine values to exclude from the domain of $(f \circ g)(x)$ in two steps.

Rules for Excluding Numbers from the Domain of $(f \circ g)(x) = f(g(x))$	**Applying the Rules to $f(x) = \frac{2}{x-1}$ and $g(x) = \frac{3}{x}$**
If x is not in the domain of g, it must not be in the domain of $f \circ g$.	Because $g(x) = \frac{3}{x}$, 0 is not in the domain of g. Thus, 0 must be excluded from the domain of $f \circ g$.
Any x for which $g(x)$ is not in the domain of f must not be in the domain of $f \circ g$.	Because $f(g(x)) = \frac{2}{g(x)-1}$, we must exclude from the domain of $f \circ g$ any x for which $g(x) = 1$. $\frac{3}{x} = 1$ Set $g(x)$ equal to 1. $3 = x$ Multiply both sides by x. 3 must be excluded from the domain of $f \circ g$.

We see that 0 and 3 must be excluded from the domain of $f \circ g$. The domain of $f \circ g$ is

$$(-\infty, 0) \cup (0, 3) \cup (3, \infty).$$