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\# 1d) $\displaystyle\int \frac{3}{\sqrt{t}}\,dt = \int 3t^{-1/2}\,dt = 3\frac{t^{1/2}}{1/2} + C$

$= 6t^{1/2} + C$

f) $\displaystyle\int (-9t^{-2} - 2t^{-1})\,dt = \int \left(-9t^{-2} - \frac{2}{t}\right)dt$

$= -\dfrac{9t^{-1}}{-1} - 2\ln|t| + C$

$= \boxed{\dfrac{9}{t} - 2\ln|t| + C}$

~~\#~~ i) $\displaystyle\int \left(\sqrt{x} + \frac{3}{x} - e^x\right)dx$

$= \dfrac{x^{3/2}}{3/2} + 3\ln|x| - e^x + C$

$= \boxed{\dfrac{2}{3}x^{3/2} + 3\ln|x| - e^x + C}$

k) $\int \sqrt{x}\,(x^2-1)\,dx = \int x^{1/2}(x^2-1)\,dx$

Don't confuse this with a $\int u\,du$!

$$= \int \left(x^{5/2} - x^{1/2}\right)dx = \frac{x^{7/2}}{7/2} - \frac{x^{3/2}}{3/2} + C$$

$$= \boxed{\frac{2}{7}x^{7/2} - \frac{2}{3}x^{3/2} + C}$$

\# 3a) $f'(x) = 1 + e^x + \frac{1}{x}$, $f(1) = 3+e$

$$f(x) = \int f'(x)\,dx = \int \left(1 + e^x + \frac{1}{x}\right)dx$$

$$f(x) = x + e^x + \ln|x| + C$$

$$\cancel{f(1) = 3+e = x + e^x + \ln|x| + C}$$

$$\underbrace{f(1) = 1 + e^1 + \ln 1 + C = 3+e}_{\text{Given initial conditions}}$$

$$C = 3 + \cancel{e} - 1 - \cancel{e} - \cancel{\ln 1}^{0} = \boxed{2}$$

So $\boxed{f(x) = x + e^x + \ln|x| + 2}$

#5a) "marg cost" is $C'(x)$, given as $5 + 2x + 1/x$. ~~Equal~~ The "total cost" (that is, $C(x)$) to produce 1 item $(x=1)$ is \$500 $(\underbrace{C(1) = 500}_{\text{init. cond.}})$

Find cost fcn. $C(x)$.

This uses the fundamental thm of calculus:

$$C(x) + \text{const} = \int C'(x)\,dx$$

$$C(x) + \text{const} = \int \left(5 + 2x + \frac{1}{x}\right) dx$$

↳ this goes over to the right

$$C(x) = 5x + x^2 + \ln|x| + \text{Const}$$

$$C(1) = 500 = 5(1) + 1^2 + \ln|1| + \text{Const}$$

$$500 = 6 + 0 + \text{Const}$$

$$\text{Const} = 494$$

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$$\boxed{C(x) = 5x + x^2 + \ln|x| + 494}$$

b) $C(20) = 5(20) + 20^2 + \ln|20| + 494$
etc.

#6) Marginal profit $(P'(x))$ for x hundred items is $P'(x) = 4 - 6x + 9x^2$

Given: $P(0) = -60$ init. condition

Find "profit fcn" $P(x)$:

$$P(x) + C = \int P'(x)\,dx$$

$$P(x) = \int (4 - 6x + 9x^2)\,dx$$

$$= 4x - \frac{6x^2}{2} + \frac{9x^3}{3} + C$$

$$= 4x - 3x^2 + 3x^3 + C$$

$$P(0) = -60 = 4(0) - 3(0^2) + 3(0^3) + C$$

$$C = -60$$

$$\boxed{P(x) = 4x - 3x^2 + 3x^3 - 60}$$

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#1 e) $\int \sqrt{4x-1}\, dx = \int (4x-1)^{1/2}\, dx$

Let $u = 4x-1$

$\frac{du}{\cancel{dx}} = 4\,dx \longrightarrow dx = \frac{du}{4}$

Substitute u-terms:

$$\int u^{1/2} \frac{du}{4} = \frac{1}{4}\int u^{1/2}\, du = \frac{1}{4}\,\frac{u^{3/2}}{3/2} + C$$

$$= \frac{2}{12}\, u^{3/2} + C = \boxed{\frac{4x-1}{6} + C}$$

f) $\int 4e^{2z}\, dz = 4\int e^{2z}\, dz$ or $2\int 2e^{2z}\, dz$

$u = 2z, \quad du = 2\,dz$

so $2\int \underbrace{2\, e^{\overbrace{2z}^{u}}\, dz}_{du}$

$$= 2\int e^{u}\, du = 2e^{u} + C = \boxed{2e^{2z} + C}$$

g) $\int \frac{2x^4}{x^5+1}\,dx$ $\quad u = x^5+1$, $\quad du = 5x^4\,dx$

$\downarrow$

$= 2\int \frac{x^4\,dx}{x^5+1}$

Almost du on top

u on bottom

Adjusted coeff

$= 2\cdot\frac{1}{5}\int \frac{5x^4\,dx}{x^5+1} = \frac{2}{5}\int \frac{du}{u}$

$= \frac{2}{5}\ln|u| + C = \boxed{\frac{2}{5}\ln|x^5+1| + C}$

☆ k) (added) $\quad \int \frac{\ln x}{x}\,dx$ $\quad u = \ln x$

$\frac{du}{\cancel{dx}} = \frac{1}{x}\,dx$

$\rightarrow \int \overset{\ln x}{u}\,\overset{\frac{dx}{x}}{du} = \frac{u^2}{2} + C = \boxed{\frac{(\ln x)^2}{2} + C}$

j) $\int \frac{e^{2x}}{e^{2x}+5}\,dx$, $\quad u = e^{2x}+5$

$du = 2e^{2x}\,dx$

$\uparrow$ Need this

$= ~~\int~~$

j continued)

$$\underbrace{\frac{1}{2}\int \frac{2\,e^{2x}\,dx}{e^{2x}+5}}_{du}$$

adjusted coeff

$$= \frac{1}{2}\int \frac{du}{u} = \frac{1}{2}\ln|u| + C$$

$$= \frac{1}{2}\ln\left|e^{2x}+5\right| + C$$

l) $$\int \frac{4x}{\sqrt{x^2+9}}\,dx = 4\int \frac{x\,dx}{(x^2+9)^{1/2}}$$ — Almost $u^n\,du$

Let $u = x^2+9$, $du = 2x\,dx$, $n = -1/2$

$$4\int x\,(x^2+9)^{-1/2}\,dx = 4\cdot\frac{1}{2}\int 2x\,(x^2+9)^{-1/2}\,dx$$

adjust coeff

$$= \cancel{4}\,2\int u^{-1/2}\,du = \frac{\cancel{4}\,2\,u^{1/2}}{1/2} + C = 8u^{1/2} + C$$

$$= \cancel{8}\,4\,(x^2+9)^{1/2} + C = \boxed{4\sqrt{x^2+9} + C}$$

n) $\displaystyle\int \frac{x}{(x+1)^2}\,dx$ Let $u = x+1$

then $x = u-1$

$dx = du$

$$= \int \frac{u-1}{u^2}\,du = \int\left(\frac{u}{u^2} - \frac{1}{u^2}\right)du$$

$$= \int\left(\frac{1}{u} - \frac{1}{u^2}\right)du$$

$$= \int \frac{1}{u}\,du - \int \frac{1}{u^2}\,du \quad u^{-2}$$

$$= \ln|u| - \frac{u^{-1}}{-1} + C$$

$$= \ln|u| + \frac{1}{u} + C$$

$$= \boxed{\ln|x+1| + \frac{1}{x+1} + C}$$