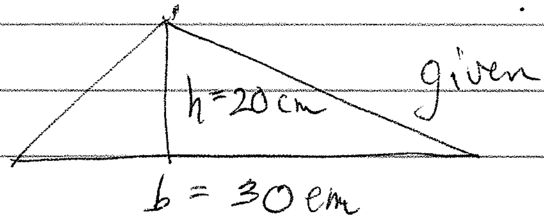
Related rate problem #14



$$\frac{db}{dt} = \frac{5\,cm}{sec}$$

$$\frac{dh}{dt} = -2\,\frac{cm}{sec}$$

given

Find $\frac{dA}{dt}$ (formula):

$A = \frac{1}{2} bh$ where all 3 variables are fcns of t.

$$\boxed{\frac{dA}{dt} = \frac{1}{2}\left(\frac{db}{dt}\cdot h + \frac{dh}{dt}\cdot b\right)}$$

Find $\left.\frac{dA}{dt}\right|_{t=3\,sec}$ (numerical answer)

Normally, a problem asks for a rate when the value of one or ~~both~~ more variables is known. This one gives a time $t = 3$ sec and you need to calculate dimensions h & b after $t = 3$, using the $\frac{dh}{dt}$, $\frac{db}{dt}$ provided. That is

$$\left.\frac{dA}{dt}\right|_{t=3} = \left.\frac{dA}{dt}\right|_{b=?,\ h=?}$$

Initial $b = 30$

$\frac{db}{dt} = 5$, so the

The base increases by $\frac{db}{dt}\cdot t$

→

since dist = rate × time: $\frac{db}{dt} \cdot t = \frac{5\,cm}{sec} \cdot 3\,sec$

change in b (lengthening) $= 15\,cm$

Hence, at $t = 3$ sec, $b = 30 + 15$ cm $= 45$ cm

Likewise, $\frac{dh}{dt} \cdot t = \frac{-2\,cm}{sec} \cdot 3\,sec = -6\,cm$

is the change in height (shortening)

So, at $t = 3$ sec, $h = 20 - 6$ cm $= 14$ cm.

Finally, $\left.\frac{dA}{dt}\right|_{t=3}$ means $\left.\frac{dA}{dt}\right|_{\substack{b=45 \\ h=14}}$

Filling everything in:

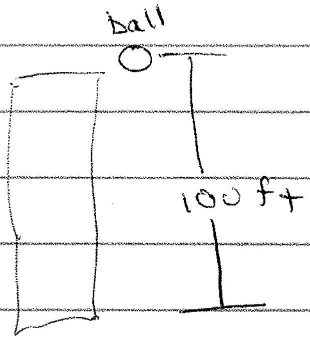
$$\frac{dA}{dt} = \frac{1}{2}\left(\frac{db}{dt} \cdot h + \frac{dh}{dt} \cdot b\right)$$

$$= \frac{1}{2}\Big(\underbrace{5 \cdot 14}_{\frac{cm \cdot cm}{sec}} + \underbrace{-2 \cdot 45}_{\frac{cm \cdot cm}{sec}}\Big) = -20\,\frac{cm^2}{sec}$$

Notice the units of dA/dt (cm^2/sec)

The (−) means area is decreasing at this rate.

Sec. 12 #9



position at time t above the ground

$s(t) = -16t^2 + 100$

notice $s(0) = 100$ ft (ball still held)

a) 2 sec after dropping:

$s(t) = -16(2^2) + 100 = \boxed{36 \text{ ft above the ground}}$

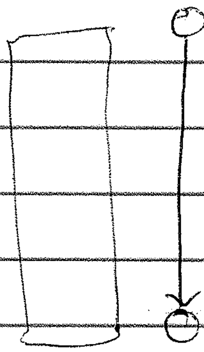
b) $\boxed{v(t) = s'(t) = -32t}$ (-32 is acceleration due to gravity in ft/sec^2)

$v(2) = -32 \frac{ft}{sec^2} \cdot 2 \text{ sec} = \boxed{-64 \frac{ft}{sec}}$

c) $a(t) = v'(t) = s''(t)$

$\boxed{a(t) = -32 \frac{ft}{sec^2}}$ Here it is, the acceleration of object in free fall

d) Added in class – how long will it take for the ball to strike the ground?



100 ft traveled represents zero ft (0 ft) above the ground.

$s(t) = 0$ when $t = ?$

Solving $0 = -16t^2 + 100$

$16t^2 = 100$

$t = 10/4$ sec

Then $v(t) = v(10/4 \text{ sec}) = -32(10/4) = -80 \frac{\text{ft}}{\text{sec}}$

The negative indicates downward travel.

(If we consider horizontal motion, negative would indicate backing up)

Marginal revenue, profit, cost — summary question

Sec 11 #6 — demand, revenue, price.

Function

$q(p) = -\frac{4(p+1)^2}{3} + \cancel{8}\ 80$, q sales per week

p price per item

What is a the weekly revenue fcn with respect to p?

$R = pq$ (price × quantity)

$R(p) = p \cdot \underbrace{q(p)}_{q \text{ is fcn of } p} = p\left(\frac{-4(p+1)^2}{3} + 80\right)$

What is the marginal revenue fcn? (w.r.t. price)

Either expand $R(p)$ or use product rule:

Expand: $R(p) = -\frac{4}{3}p(p+1)(p+1) + 80p$

$$= -\frac{4}{3}p(p^2 + 2p + 1) + 80p$$

$$= -\frac{4}{3}\left(p^3 + 2p^2 + p\right) + 80p$$

$$R'(p) = -\frac{4}{3}\left(3p^2 + 4p + 1\right) + 80$$

$$= -4p^2 - \frac{16}{3}p - \frac{4}{3} + 80$$

$R'(p)$ gives the ~~[illegible]~~ change in revenue for each <u>additional</u> <u>dollar</u> <u>charged</u> ($\$1$ increase in price)

$$R'(4) = -4(4^2) - \frac{16}{3}(4) - \frac{4}{3} + 80$$

$$= -64 - \frac{64}{3} - \frac{4}{3} + 80$$

$$= 16 - \frac{68}{3} = \frac{48 - 68}{3} = -\frac{20}{3}$$

That is, revenue falls by $\$20/3$ ~~[illegible]~~ ~~[illegible]~~ if price is increased from $\$4$ to $\$5$ per item.