Math 220-02 Spring '16 Quiz #2 Key

Find the indicated limits. Show your work where applicable.

$\lim_{x\to 2}\frac{x^2-x-6}{x-3} = \frac{4-2-6}{2-3} = \frac{-4}{-1} = \boxed{4}$

$\lim_{x\to 3}\frac{x^2-x-6}{x-3} = \frac{9-3-6}{3-3} = \frac{0}{0}$; need algebra: $\frac{\cancel{(x-3)}(x+2)}{\cancel{(x-3)}}$ $\lim_{x\to 3}(x+2) = \boxed{5}$

Const / zero some kind of ∞

$\lim_{x\to 4^-}\frac{x-1}{x-4}$ Inspect values close to 4 on left. Conclude LHL $= -\infty$

x	f(x)
3.8	-30
3.9	-300
3.99	$\downarrow -\infty$

$\lim_{x\to 4^+}\frac{x-1}{x-4}$ Inspect values close to 4 on right. Conclude RHL $= +\infty$

x	f(x)
4.1	30
4.01	300
	$\downarrow +\infty$

$\lim_{x\to 4}\frac{x-1}{x-4}$ so, since LHL $\neq$ RHL, $\lim_{x\to 4} f(x)$ DNE

LCD $5x$

$\lim_{x\to 5}\frac{\frac{1}{x}-\frac{1}{5}}{x^2-25} = \frac{0}{0}$; need algebra: $\frac{-1\cancel{(5-x)}}{5x}\cdot\frac{1}{\cancel{(x-5)}(x+5)} = \frac{-1}{5x(x+5)}$

$\lim_{x\to 5}\frac{-1}{5x(x+5)} = \boxed{\frac{-1}{250}}$

2. Determine whether the piecewise function is continuous or not at the indicated values of x. If it is not continuous, give a thorough explanation *using the definition of "continuity at x = a."*

$$f(x)=\begin{cases}\sqrt{x-2}, & \text{if } x<3\\ x-2, & \text{if } 3\le x<6\\ \frac{2x}{3}, & \text{if } x>6\end{cases}$$

HOTSPOTS: $x=3$ & $x=6$

Inspect LHL: $\lim_{x\to 3^-} f(x) = \sqrt{3-2} = \sqrt{1} = 1$

$\parallel$

RHL: $\lim_{x\to 3^+} f(x) = 3-2 = 1$

AND $f(3) = 1 =$ limit

Hence, $\lim_{x\to 3} f(x) = 1$,

fcn is cts at $x=3$

$f(6)$ is not defined, so fcn is not cts at $x=6$

However: LHL: $6-2=4$ = RHL: $\frac{2(6)}{3} = 4$ Think RHL = LHL makes it cts.

$P(x) = R(x) - C(x)$, x = units sold or produced

$C(x) = mx + b$, $R(x) = px$

3. A blue jeans manufacturer figures that the general cost of operating the machines for a day is $120, fixed, "b" in $mx+b$
regardless of how many jeans are made that day. Each pair of jeans will cost about $14 to make (for cost of materials, labor, etc.). The selling price for each pair will be $28. $= p$

variable, "m" in $C(x) = mx + b$

How many jeans must be sold each day in order for the manufacturer to break even?

$P(x) = 0$

$$P(x) = R(x) - C(x)$$

$$= 28x - (14x + 120) = 0 \rightarrow x = 8.57 \text{ units}$$

$$\Rightarrow x = 9 \text{ pairs}$$

$= b$

Suppose the daily cost of running the machinery goes up to $200. What price will the manufacture have to charge per pair to make a profit if 10 pairs of jeans are sold each day? (Round up to the nearest dollar.)

$x = 10$

$p = ?$

$$P(x) > 0$$

$$P(10) = \cancel{14(10)}$$

$$= 10p - (14 \cdot 10 + 200) > 0$$

$$= 10p - 340 > 0$$

$$p > \frac{340}{10} = \$34$$

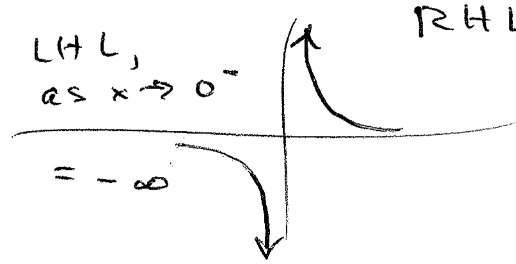
$$p = \$35$$

$$f(x) = \frac{1-x}{1+x}$$

$$\text{DNE} = \lim_{x \to 0} \frac{1}{x}$$

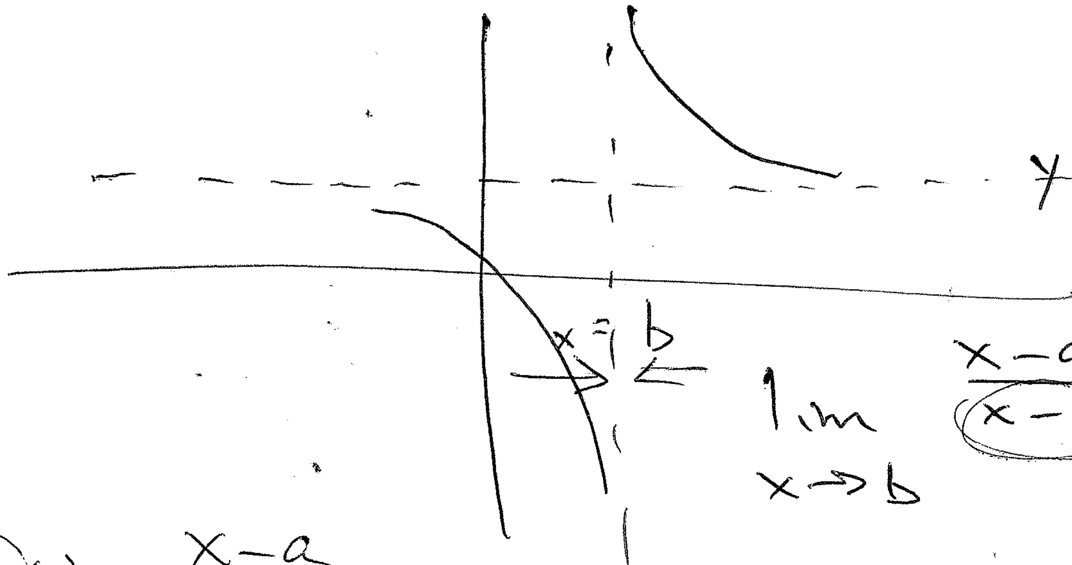
LHL, as $x \to 0^-$ $= -\infty$

RHL, as $x \to 0^+$ $= +\infty$



LHL $\neq$ RHL

$$f(x) = \cancel{\frac{a-x}{b-x}} \quad \frac{x-a}{x-b}$$



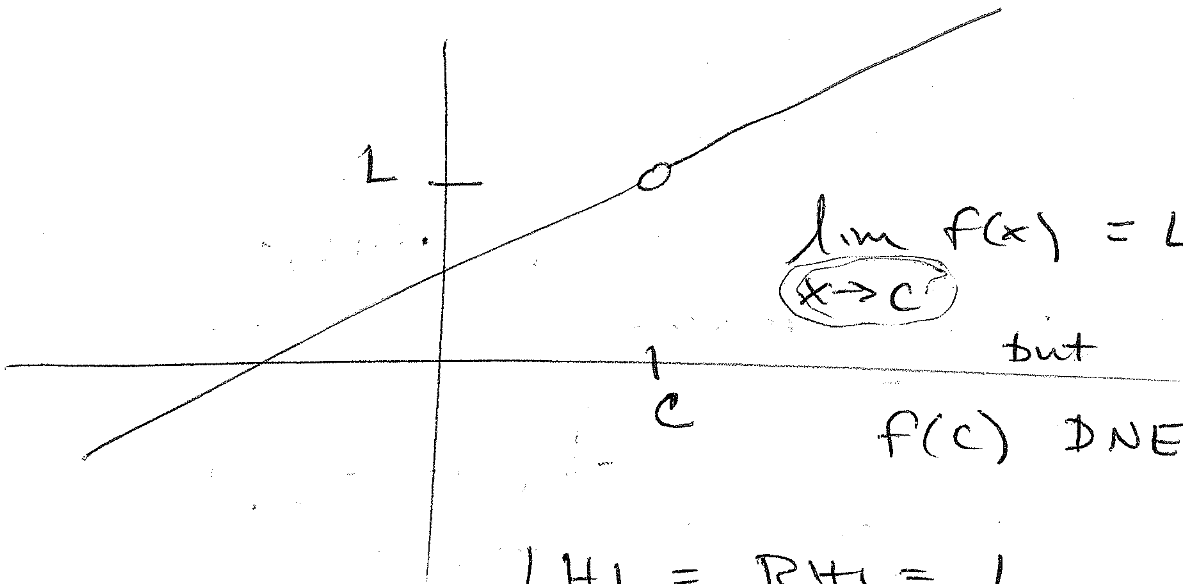
$$\lim_{x \to b} \frac{x-a}{x-b} = ?$$

$$f(x) = \frac{x-a}{x-b}$$

is candidate for LHL, RHL analysis when $\lim_{x \to b} f(x)$

$x - b > 0$, $b < x$

$x - b < 0$, $b > x$



$$\lim_{x \to c} f(x) = L$$

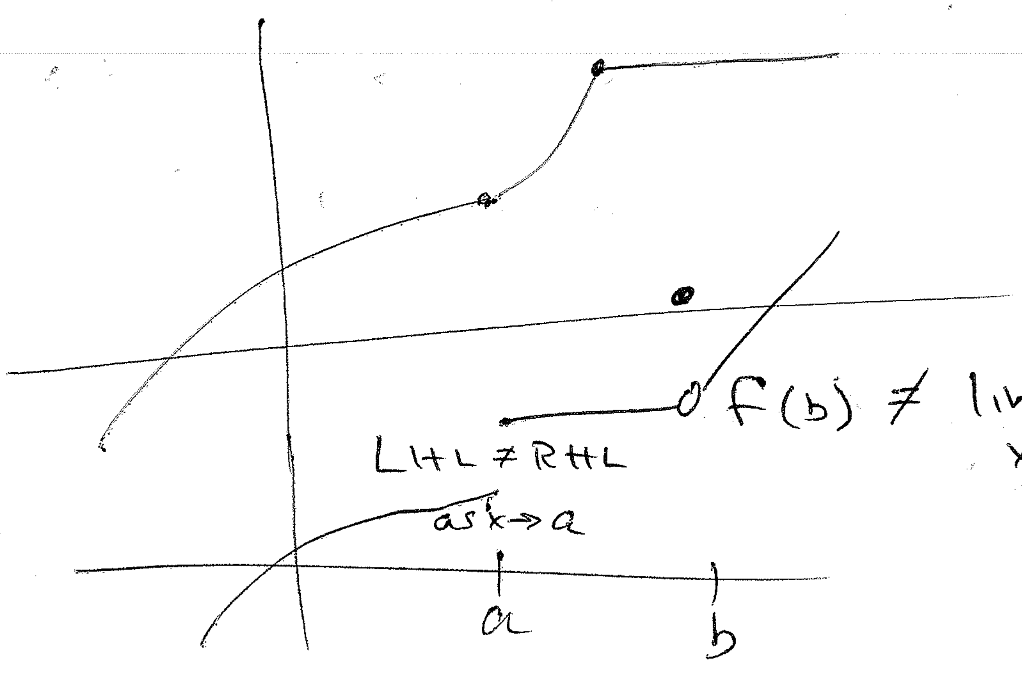
but

$f(c)$ DNE

LHL = RHL = L

$$\frac{a-b}{b-a} = -1$$

So $\dfrac{x-5}{5-x} = \dfrac{-1(-x+5)}{(5-x)}$



$f(b) \neq \lim\limits_{x \to b}$

LHL ≠ RHL as $x \to a$