MATH 220 QUIZ #2 (TAKE-HOME) Key

You may use your book and notes to work on this, but do not work with another student or with a tutor or other mentor. Do not use a calculator.

Find the indicated limits:

1. $\lim_{x\to -1} 2x^3 - 4x + 7 =$ 9 $\qquad \lim_{x\to -4} \frac{x^2-16}{x+4} =$ −8

$\lim_{x\to 7^-} \frac{3}{x-7} =$ $-\infty$ $\qquad \lim_{x\to 7^+} \frac{3}{x-7} =$ ∞ So $\lim_{x\to 7} \frac{3}{x-7}$ DNE

2. Given $f(x) = \begin{cases} 1, & \text{if } x \text{ is an integer} \\ -1, & \text{otherwise} \end{cases}$

$\lim_{x\to 1/2} f(x) =$ −1 $\qquad \lim_{x\to 0} f(x) =$ DNE (hard one to show)

3. Given $g(x) = \begin{cases} 1-x^2, & \text{if } x \le 0 \\ x+2, & \text{if } 0 < x \le 4 \\ 10-x, & \text{if } x > 4 \end{cases}$

$\lim_{x\to 0^-} g(x) =$ 1 $\qquad \lim_{x\to 0^+} g(x) =$ 2 $\qquad \lim_{x\to 0} f(x) =$ DNE

$\lim_{x\to 2^-} g(x) =$ 4 $\qquad \lim_{x\to 2^+} g(x) =$ 4 $\qquad \lim_{x\to 2} g(x) =$ 4

4. Find the break-even production amount for a manufacturing operation whose cost function is $C(x) = 20x + 320$ when the goods are to be sold for \$15 each.

Find x such that: $\underbrace{20x + 320}_{\text{cost}} = \underbrace{15x}_{\text{revenue}} \implies$ i.e. Profit = 0 when Cost = Revenue

$x = -64$ is soln, which is unfeasible. (over)

5. Show all steps to find the value of the slope of the tangent to the curve of function $f(x) = \frac{2}{x}$ at $x = 1$.

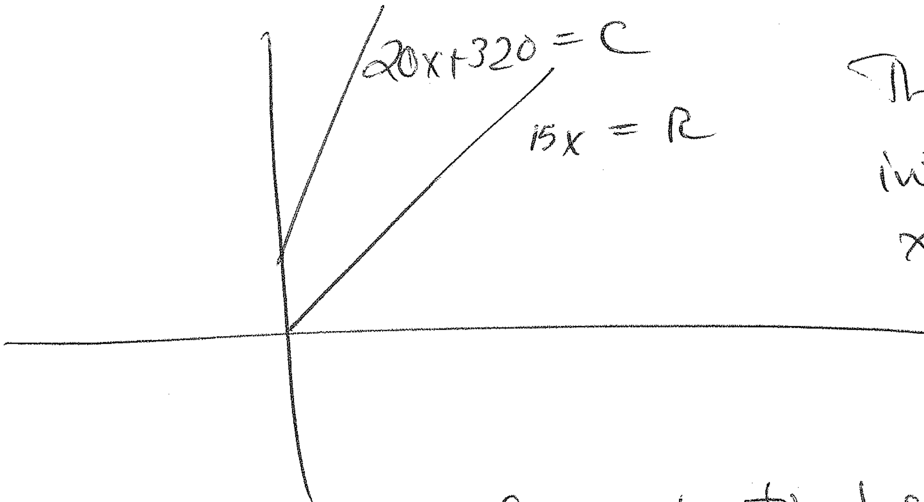
$$\lim_{h\to 0} \frac{f(1+h) - f(1)}{h} = \lim_{h\to 0} \frac{\frac{2}{1+h} - \frac{2}{1}}{h} = \lim_{h\to 0} \frac{2-2-\cancel{2h}}{h} = \boxed{-2}$$

What is the function that gives the slope of the tangent for ***any*** x on this curve? (That is, what is the general expression for the difference quotient?)

$$\lim_{h\to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h} = \lim_{h\to 0} \frac{\frac{x-(x+h)}{x(x+h)}}{h} = \lim_{h\to 0} \frac{-h}{hx(x+h)}$$

$$= \lim_{h\to 0} \frac{-1}{x(x+h)} = \boxed{-\frac{1}{x^2}}$$

Notice the slope of revenue fcn is 15
+ the slope of cost fcn is 20



They don't intersect for $x > 0$.

We need slope of cost to be lower than slope of revenue fcn.

Try charging \$30 per item.

Then $20x + 320 = 30x$

$x = 32$ items

is Break even pt.

Math 220, Quiz 2 Key (detailed)

Quiz 2 - Take home: — Solutions

1. $\lim_{x \to -1} 2x^3 - 4x + 7 = 2(-1)^3 - 4(-1) + 7 = 9$

Because $\lim_{x \to a} P(x) = P(a)$ when $P(x)$ is a polynomial.

$\lim_{x \to -4} \frac{x^2-16}{x+4} = \lim_{x \to -4} (x-4) = -8$?

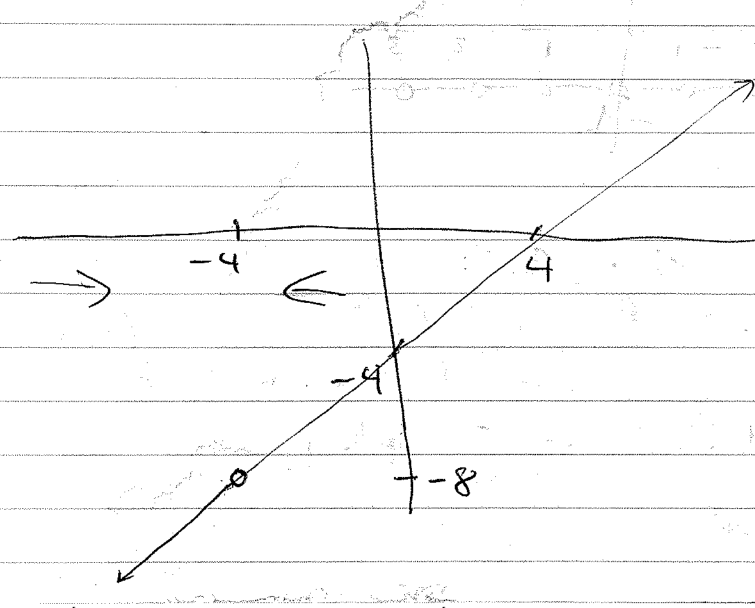
Notice the natural domain of the ~~expression~~ function

$f(x) = \frac{x^2-16}{x+4}$ is $x \in \mathbb{R}$ s.t. $x \neq -4$.

~~So the (value of (f(x) at x = -4 DN~~

That is, $f(-4)$ is not defined.

But the limit as $x \to -4$, as the graph shows, does exist. Clearly, it is -8.



$$\lim_{x \to 7^-} \frac{3}{x-7} = \frac{3}{\text{tiny negative values}} = \text{unboundedly large negative values}$$

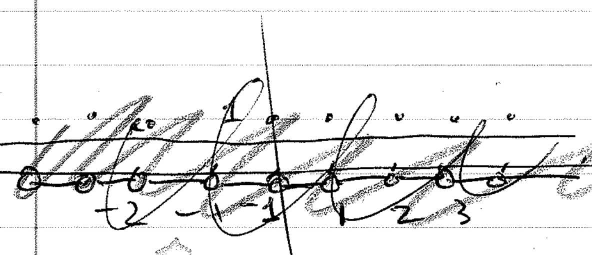
or $-\infty$

$$\lim_{x \to 7^+} \frac{3}{x-7} = \frac{3}{\text{tiny positive values}} = \text{unboundedly large positive values}$$

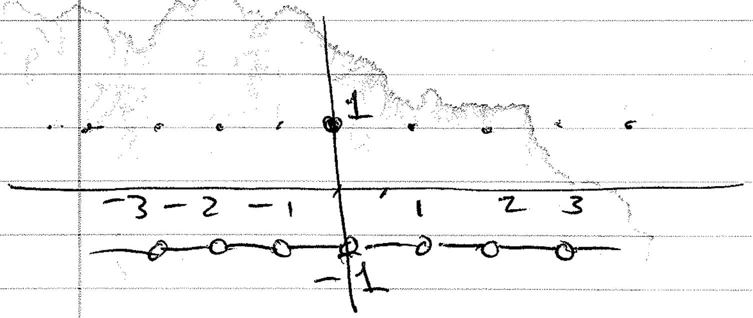
or ∞

So, since LHL $\neq$ RHL, $\lim_{x \to 7}$ DNE

2. Given $f(x) = \begin{cases} 1 , & x \in \mathbb{Z} \\ -1 , & \text{otherwise} \end{cases}$



~~$\lim_{x \to 1/2} f(x) =$~~



$\lim_{x \to 1/2} f(x) = -1$

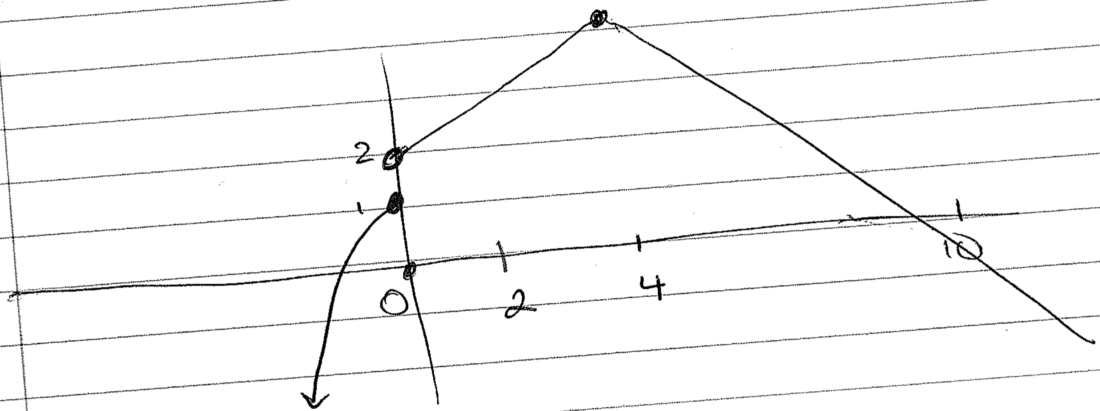
$\lim_{x \to 0} f(x)$ DNE

Since the graph jumps from -1 to 1 (not a gradual approach)

This was actually a hard problem. If you could even sketch the graph, good for you.

3. $g(x) = \begin{cases} 1-x^2, & x \le 0 \\ x+2, & 0 < x \le 4 \\ 10-x, & x > 4 \end{cases}$

Check the values of $g(x)$ at the domain interval end pts.



The graph of this piecewise fcn is seen.
I'm interested in your ability to sketch
a line or parabola, in so far as
x + y intercepts + at any other values
of x that will help you see the picture

4. Break evenpoint: where $P(x) = 0$
That is, where $R(x) = C(x)$,

since $P(x) = R(x) - C(x)$

$R(x) = 15x$ from given price of $1

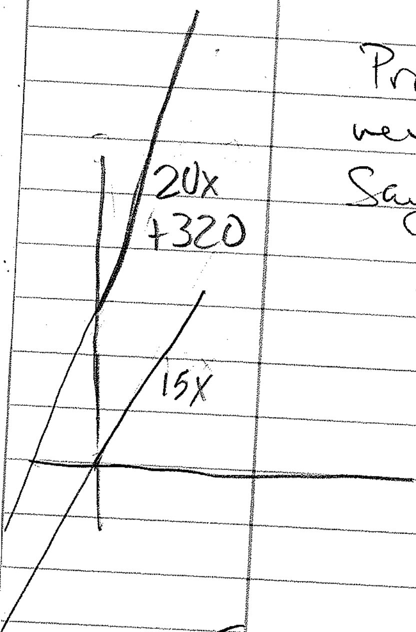
$\cancel{P(x) = 20x + 320 - 15x = 5x + 320}$

$\cancel{P(x) = 5x + 320 = 0 \text{ when } x =}$

$P(x) = R(x) - C(x) = 15x - (20x$

$-5x - 320$ when $x =$

$P(x) =$



This doesn't make sense, We need $x > 0$

Price \$15 is not high enough. We'll never break even. The lines don't intersect. Say price = \$30. Then:

$$R(x) = 30x \quad , \quad P(x) = 30x - 20x - 320 = 0$$

$$10x = 320$$

$$x = 32$$ items must be sold to break even

5. $f(x) = \dfrac{2}{x}$, Diff Quotient:

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{2}{x+h} - \frac{2}{x}}{h} = \lim_{h \to 0} \frac{\frac{2x - 2x - 2h}{x(x+h)}}{h}$$

$$\cancel{= \lim_{h \to 0} \frac{-2h}{x^2h + xh}} = \lim_{h \to 0} \frac{-2\cancel{h}}{x\cancel{h}(x+h)} = \frac{-2}{x^2}$$

Then, at $x = 1$, slope to $f(x) = \dfrac{-2}{(1)^2} = -2$

Finally, the slope of the tangent at any x is the derivative of $f(x)$.

$$f(x) = \frac{2}{x} \qquad f(x+h) = \frac{2}{x+h}$$

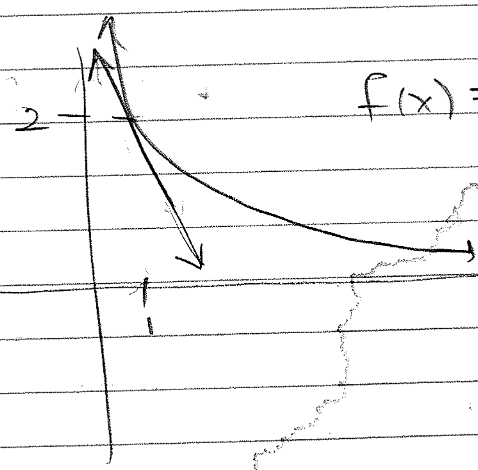
$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\frac{2}{x+h} - \frac{2}{x}}{h}$$

$$= \lim_{h\to 0} \frac{\frac{2x-2(x+h)}{x(x+h)}}{h} = \lim_{h\to 0} \frac{2x-2x-2h}{x(x+h)h}$$

$$= \lim_{h\to 0} \frac{-2h}{x^2h+xh^2} = \lim_{h\to 0} \frac{-2}{x^2+xh}$$

$$= \frac{\lim_{h\to 0} -2}{\lim_{h\to 0} x^2+xh} = \boxed{\frac{-2}{x^2} = f'(x)}$$

Also, $f'(1) = \frac{-2}{1^2} = \boxed{-2}$ slope of tangent at $x=1$



$f(x) = \frac{2}{x}$ on $(0, \infty)$

half the natural domain