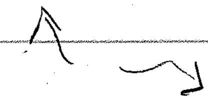
Quiz 9 Key — Curve Sketching

$g(x) = -\frac{4}{3}x^3 + 2x + 5$

shape



Dom: $\mathbb{R}$ $(-\infty, \infty)$ like all polynomials

$g(0) = 5$, $(0,5)$ y-int $\qquad g(x) = 0$ someplace!

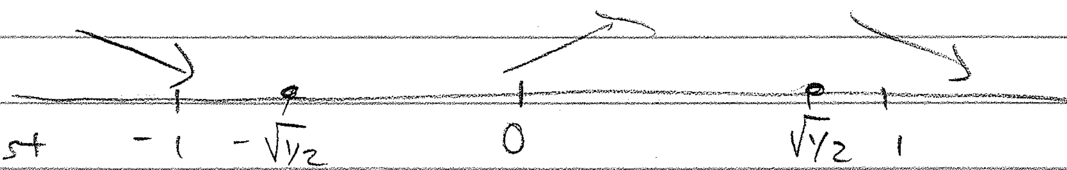
Critical numbers: $g'(x) = 0$ at $x = \pm\sqrt{2}/2$

from $g'(x) = -4x^2 + 2 = 0 \longrightarrow 2 = 4x^2$

$\longrightarrow x^2 = \frac{2}{4} = \frac{1}{2} \longrightarrow x = \pm\sqrt{\frac{1}{2}}$

$\sqrt{1/2} = 1/\sqrt{2} \approx 1/1.414 = .707$

FDT



Values to test $-1, 0, 1$ into $g'(x)$ for sign

$g'(-1) = -4+2 < 0 \qquad g'(1) = -4+2 < 0$

g decreasing on $(-\infty, -\sqrt{1/2}) \cup (\sqrt{1/2}, \infty)$

$g'(0) = 2 > 0$, g increasing on $(-\sqrt{1/2}, \sqrt{1/2})$

SDT If $g''(c) > 0$ then $g(c)$ is local min

If $g''(c) < 0$ then $g(c)$ is local max

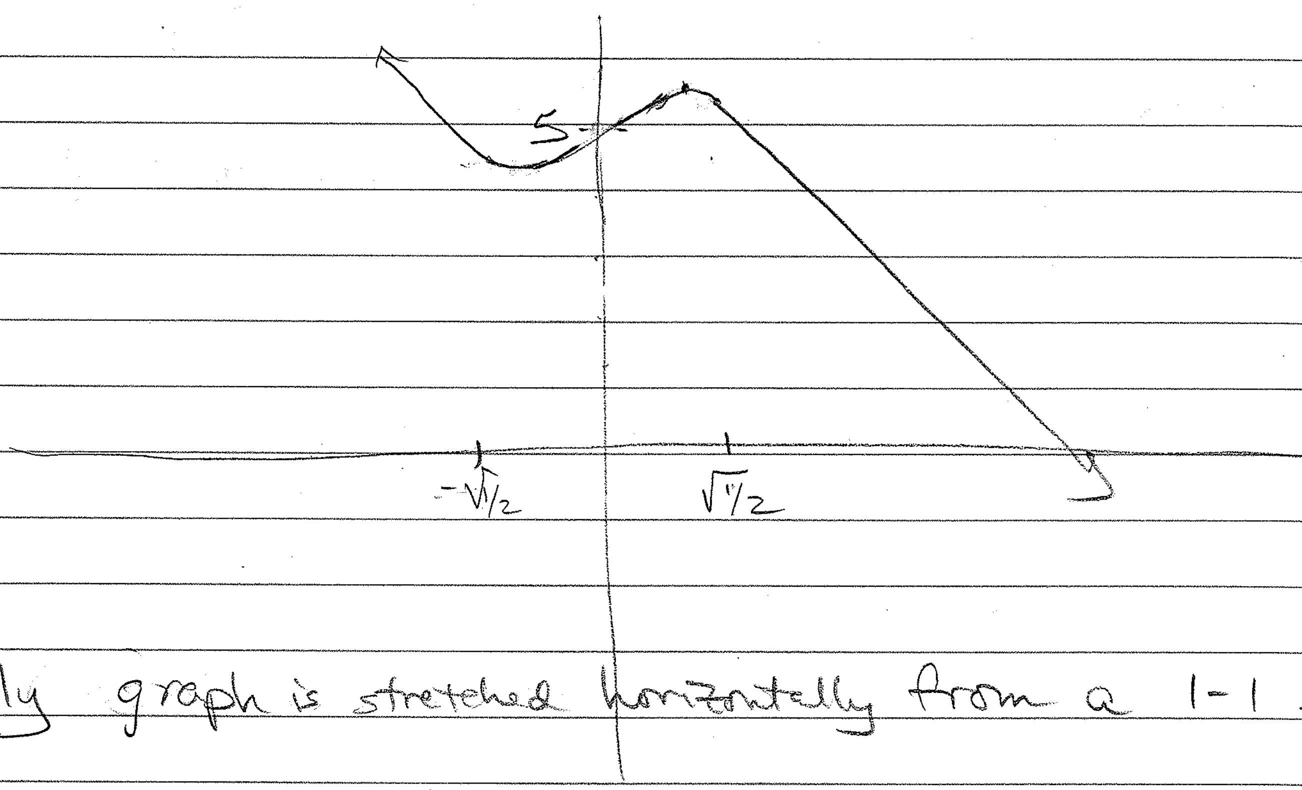
If $g''(c) = 0$ possible POI

$g''(x) = -8x$, $g''(-\sqrt{1/2}) > 0$, so $g(-\sqrt{1/2})$ is local max

$g''(\sqrt{1/2}) \geq 0$ so $g(\sqrt{1/2})$ is local min.

Where is $g''(x) = 0$, if anywhere?

$g''(x) = -8x = 0$ at $x = 0$. We know from FDT above that g increases on $(-\sqrt{\frac{1}{2}}, \sqrt{\frac{1}{2}})$ and from SDT concavity changed from $-\sqrt{1/2}$ to $\sqrt{1/2}$, so we have enough to conclude (more than enough) that $(0,5)$ is a POI.



My graph is stretched horizontally from a 1-1 scale.