Thurs Sept 7 notes

Over-view of work so far → Sec. 5.3 - continuation of 1.6, 2.2, then 5.3

where 1.6 contains rational eqns, radical eqns, and quadratic eqns

2.2 & 5.3 are just quadratic.

[Aside - Fri mini-quiz - solve radical eqn + check answers ("extraneous solns.")

- solve a quadratic by completing square

Look at not only how to get solutions, but what the solutions indicate about the parabola the equation represents.

Three things can happen:

There are TWO examples of I

I

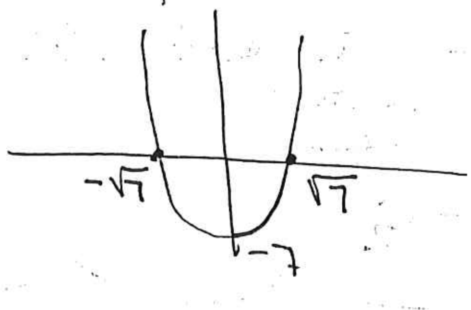
$x^2 = 7$

$\sqrt{x^2} = \pm\sqrt{7}$

$x = \pm\sqrt{7}$

2 solns to

$y = x^2 - 7$, a parabola



I

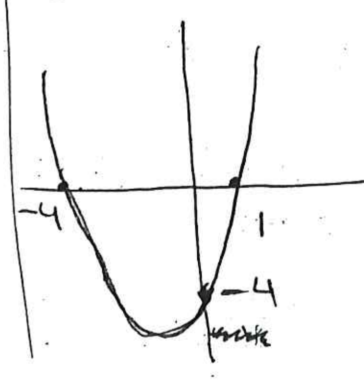
$x^2 + 3x - 4 = 0$

$(x-1)(x+4) = 0$

$x = 1, -4$

2 solns to

$y = x^2 + 3x - 4$



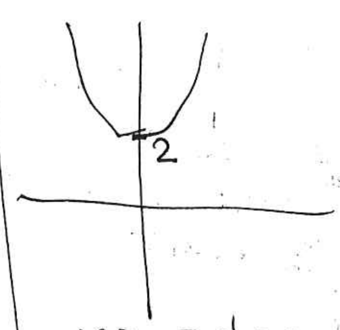
II

$x^2 + 2 = 0$

$x^2 = -2$

$\sqrt{x^2} = \pm\sqrt{-2}$

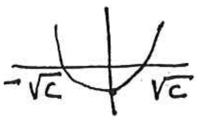
$x = \pm\sqrt{-2}$



no solns

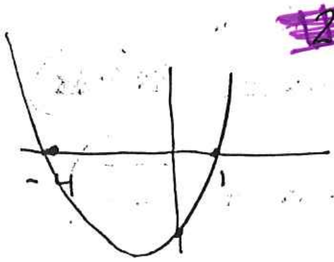
III (continued →)

Solve quadratic eqns. $y = x^2 + bx + c$



I $\quad y = x^2 = c \rightarrow x = \pm\sqrt{c}$

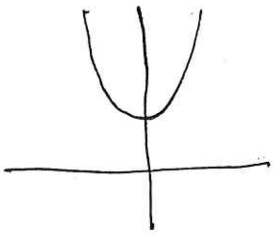
2 solns.



~~2~~ I $\quad y = x^2 + 3x - 4 = 0 \rightarrow (x-1)(x+4) = 0$

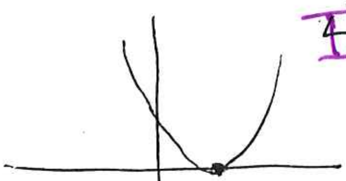
$\rightarrow x = 1, -4$

2 solns.



3 II $\quad y = x^2 + 2 = 0 \longrightarrow$ does not factor in the reals

no solns.



"double root"

~~4~~ III $\quad y = x^2 - 2x + 1 = (x-1)(x-1)$

$= (x-1)^2 = 0$

$x = 1 \quad$ 1 soln.

from previous page.

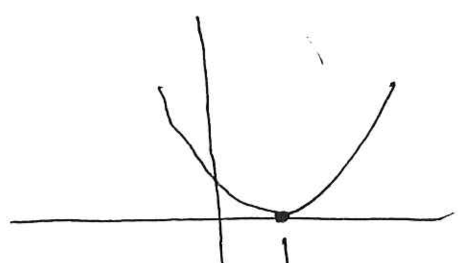
III

$x^2 - 2x + 1 = 0$

$(x-1)(x-1) = 0$

$(x-1)^2 = 0$

$x = 1$ one soln.



"double root"

Wksht #15 $5k^2 = 60 - 20k$

$5k^2 = 60 - 20k$

$\cancel{5}(k^2) = \cancel{5}(12 - 4k)$

$$k^2 = 12 - 4k$$

$$k^2 + 4k + 4 = 12 + 4$$

$\frac{4}{2}$

$$(k+2)^2 = 16$$

$$k + 2 = \pm 4$$

$$k = -2 + 4 = 2$$

$$-2 - 4 = -6$$

Equivalent words

"roots" $\longrightarrow$ "zeros" $\longrightarrow$ "solutions"

$\longrightarrow$ "x-intercepts"

Easy example : $x^2 + 8x - 9 = 0$

$(x-1)(x+9) = 0$, $x = 1, -9$

The form is $(x - r_1)(x - r_2) = f(x)$

where r_1, r_2 are roots

and therefore $f(r_1) = 0$ & $f(r_2) = 0$

Equivalent Statements r is a root of $f(x)$ $\Longleftrightarrow$ $x - r$ is a factor of $f(x)$ $\Longleftrightarrow$ $f(r) = 0$

$\Longleftrightarrow$ $\frac{f(x)}{x - r_1} = x - r_2$

Number factoring is analogous to polynomial factoring:

$$38 = (19)(2) \quad \text{so} \quad \frac{38}{19} = 2$$, no remainder

$$x^2 - 4 = (x-2)(x+2) \quad \text{so} \quad \frac{x^2-4}{x-2} = x+2$$

notice - there is no remainder

More Complete the square

New ex $8x^2 + 16x = 42$

reduce by 2 $\quad 4x^2 + 8x = 21$

Completing the square

$$4(x^2 + 2x + 1) = 21 + 4$$

$$4(x+1)^2 = 25$$

$$(x+1)^2 = \frac{25}{4}$$

$$x+1 = \pm\sqrt{\frac{25}{4}}$$

$$x = -1 \pm \frac{5}{2} = \boxed{\frac{3}{2} \text{ \& } -\frac{7}{2}}$$

2 roots

What's happening when we complete a square?

$$\underbrace{1x^2 + bx}_{\text{Given}} + \underbrace{\boxed{\tfrac{b}{2}}^2}_{\text{Step to complete the square}} = \underbrace{\left(x + \boxed{\tfrac{b}{2}}\right)^2}_{\text{This is the square we completed.}}$$

To solve by Quadratic Formula (QF) a Quad Eqn (QE)

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

QF is the soln to to QE $y=ax^2+bx+c$

The b^2-4ac is the discriminant.

This tells us how many solns we have to a QE, Q Function $y=f(x)$

Three things can happen!

- If $b^2-4ac>0$, then two solns.
- If $b^2-4ac<0$, then zero solns.
- If $b^2-4ac=0$, then one soln.

Ex $y=2x^2-5x+2$

Discriminant $b^2-4ac = 25-4(2)(2)=9>0$

so there are two solutions (zeros, roots, x-intercepts) to $y=2x^2-5x+2$

Solve: By QF: $x = \frac{5 \pm \sqrt{25-4(2)(2)}}{2(2)} = \frac{5 \pm \sqrt{9}}{4}$

$$x = \frac{5 \pm 3}{4} = 2, \frac{1}{2}$$