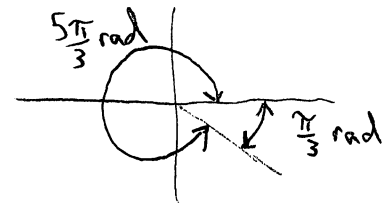


## Solution set

$$\textcircled{1.} \quad \theta = 300^\circ = 300^\circ \left( \frac{\pi \text{ radians}}{180^\circ} \right) = \frac{5\pi}{3} \text{ radians.}$$

$$\begin{aligned} \sin\left(\frac{5\pi}{3}\right) &= -\sin\left(\frac{\pi}{3}\right) \\ &= -\frac{\sqrt{3}}{2}. \end{aligned}$$



$$\cos\left(\frac{5\pi}{3}\right) = \cos\left(\frac{\pi}{3}\right) = \frac{1}{2}.$$

$$\tan\left(\frac{5\pi}{3}\right) = \frac{\sin\left(\frac{5\pi}{3}\right)}{\cos\left(\frac{5\pi}{3}\right)} = \frac{-\frac{\sqrt{3}}{2}}{\frac{1}{2}} = -\frac{\sqrt{3}}{2} \cdot \frac{2}{1} = -\sqrt{3}.$$

$$\begin{aligned} \textcircled{2.} \quad 2 \ln(x) + 3 \ln(y) - \ln(z) &= \ln(x^2) + \ln(y^3) - \ln(z) \\ &= \ln\left(\frac{x^2 y^3}{z}\right). \end{aligned}$$

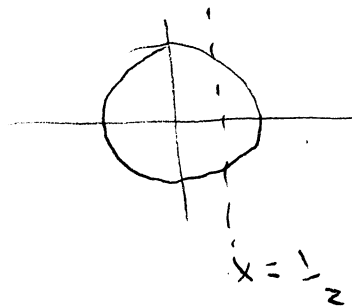
$$\textcircled{3.} \quad \text{a.) } x = \log_8 2 \text{ means } 8^x = 2, \text{ so } x = \frac{1}{3}.$$

$$\begin{aligned} \text{b.) } \log_5 \frac{1}{125} &= \log_5 (125^{-1}) \\ &= -\log_5 (125) \\ &= -\log_5 (5^3) \\ &= -3 \log_5 (5) \\ &= -3 \cdot 1 \\ &= -3. \end{aligned}$$



$$(5.) (a.) 2 \cos x - 1 = 0$$

$$\Rightarrow \cos x = \frac{1}{2}$$



So  $x$  is an angle whose

cosine is  $\frac{1}{2}$ . In quadrant 1,

we find  $x = \frac{\pi}{3}$  radians. In quadrant 4, we

find  $x = \frac{5\pi}{3}$  radians. So,  $x = \frac{\pi}{3}$  or  $x = \frac{5\pi}{3}$

radians.

$$(b.) e^{7-4x} = 6$$

(take  $\ln$  of both sides)

$$\Rightarrow 7 - 4x = \ln 6$$

$$\Rightarrow -4x = \ln 6 - 7$$

$$\Rightarrow x = \frac{\ln 6 - 7}{-4} = \frac{7 - \ln 6}{4}$$

$$(c.) e^{e^x} = 10$$

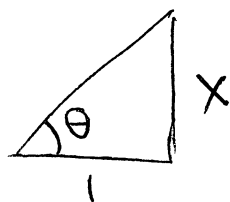
(take  $\ln$  of both sides)

$$\Rightarrow e^x = \ln 10$$

(do it again)

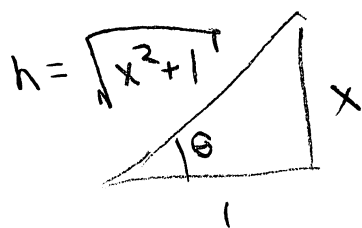
$$\Rightarrow x = \ln(\ln 10)$$

⑥.  $\tan^{-1}(x)$  is an angle. Let's call the angle  $\Theta$ . So  $\Theta = \tan^{-1}(x)$ . In other words,  $\Theta$  is an angle whose tangent is  $x$ , or equivalently,  $\frac{x}{1}$ . We can fill-in a triangle with this info:



and we can use the Pythagorean Theorem to get the hypotenuse:

$$h^2 = x^2 + 1^2$$



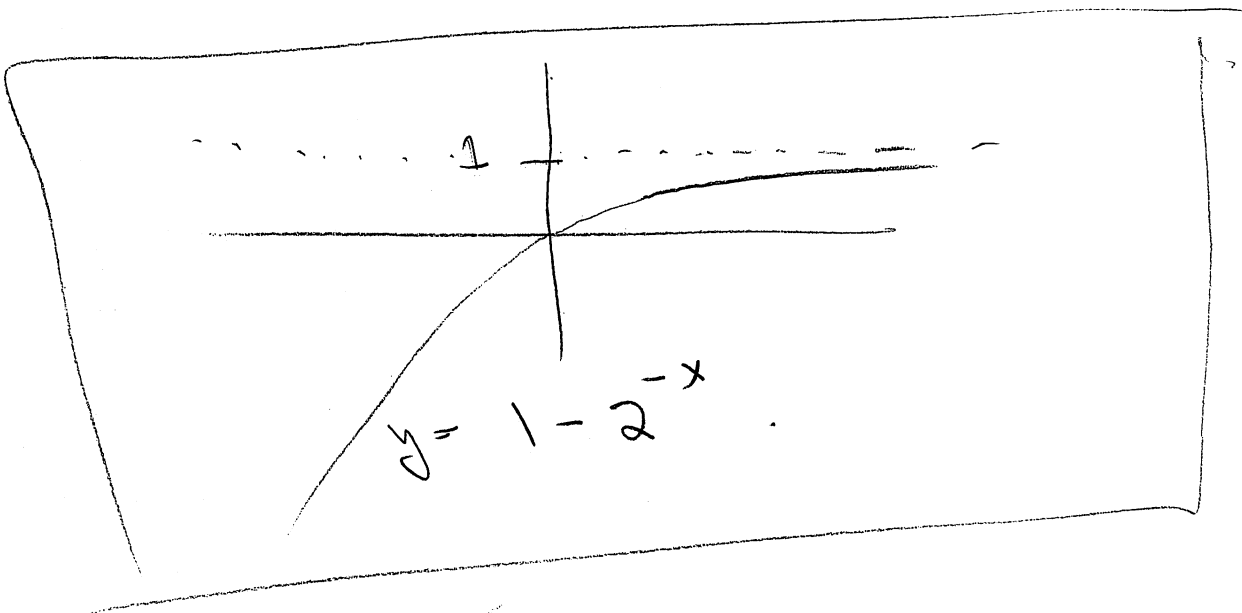
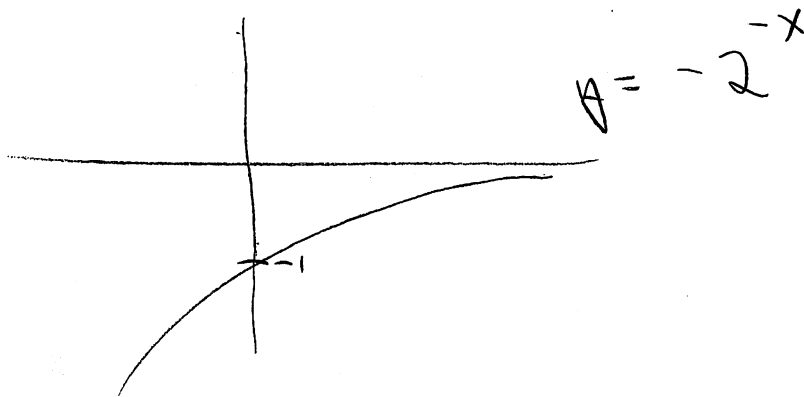
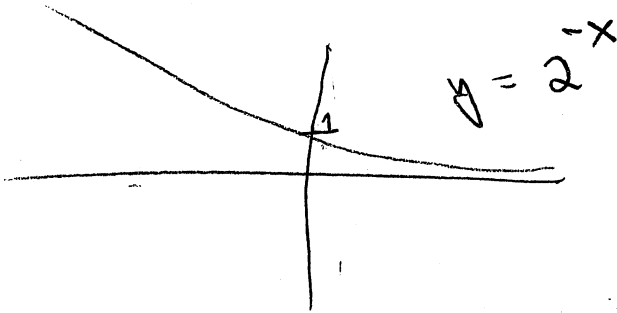
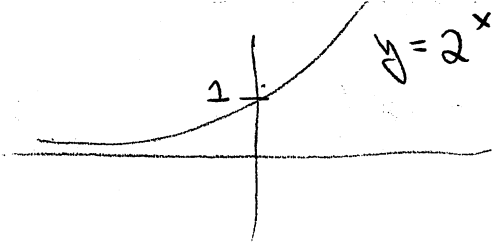
$$\Rightarrow h = \pm \sqrt{x^2 + 1} \quad \text{and}$$

we use only positive root since  $h$  is a distance. From the triangle, we now just write down

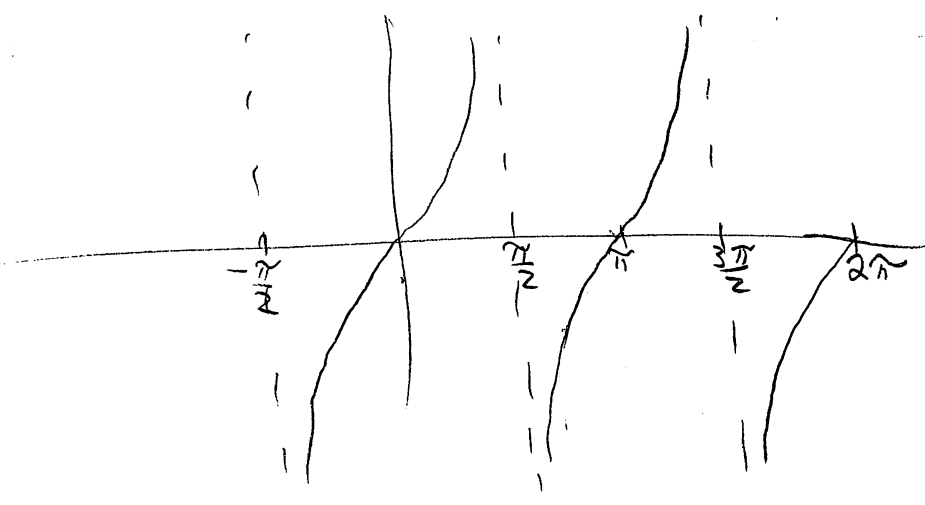
$$\sin \Theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{x}{\sqrt{x^2 + 1}}$$

$$\text{So } \sin(\tan^{-1} x) = \sin \Theta = \frac{x}{\sqrt{x^2 + 1}}$$

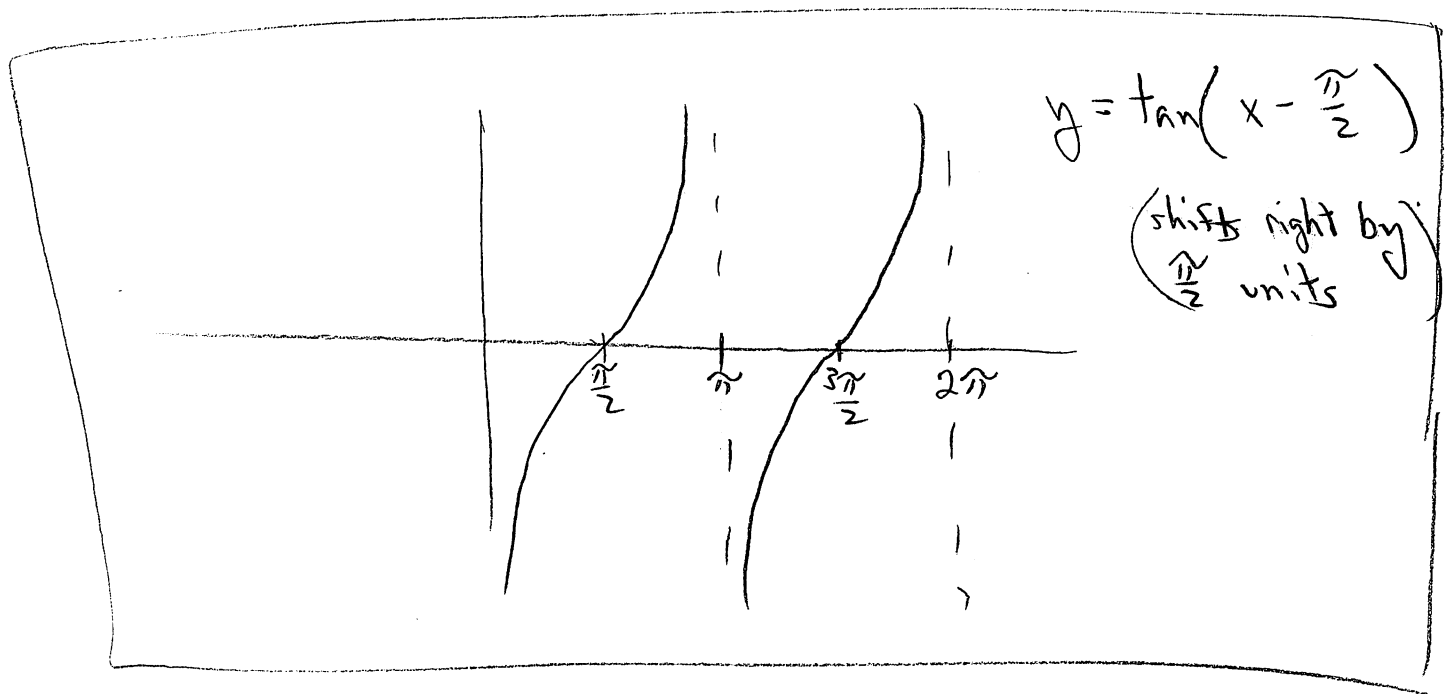
7 (5)



7 (b)



$$y = \tan x$$



$$y = \tan\left(x - \frac{\pi}{2}\right)$$

(shift right by  $\frac{\pi}{2}$  units)

8. All three pieces are polynomials, which are continuous everywhere, so discontinuities can only happen at  $x=0$  and  $x=1$ . Let's check each location.

$$\underline{x=0} : \lim_{x \rightarrow 0^-} f(x) = 0 + 2 = 2$$

$$\lim_{x \rightarrow 0^+} f(x) = 2(0)^2 = 0$$

$$\underline{x=1} : \lim_{x \rightarrow 1^-} f(x) = 2(1)^2 = 2$$

$$\lim_{x \rightarrow 1^+} f(x) = 3 - 1 = 2.$$

The two-sided limit of  $f(x)$  does not exist at  $x=0$  because  $\lim_{x \rightarrow 0^-} f(x) \neq \lim_{x \rightarrow 0^+} f(x)$ . So  $f(x)$  is not continuous at  $x=0$ .

The two-sided limit of  $f(x)$  exists at  $x=1$  because  $\lim_{x \rightarrow 1^-} f(x) = 2 = \lim_{x \rightarrow 1^+} f(x)$ . And the value

of  $\lim_{x \rightarrow 1} f(x)$  is equal to  $f(1)$ . So  $f(x)$  is is

continuous at  $x=1$ .

Hence,  $f(x)$  is discontinuous only at  $x=0$ .

$$\textcircled{9} \text{ (a.) } \lim_{x \rightarrow 3} \frac{x^2 - 9}{x^2 - x - 6} = \lim_{x \rightarrow 3} \frac{(x+3)\cancel{(x-3)}}{\cancel{(x-3)}(x+2)}$$

$$= \lim_{x \rightarrow 3} \frac{x+3}{x+2} = \frac{6}{5} .$$

$$\text{(b.) } \lim_{x \rightarrow 4^-} \frac{x^2 - 16}{|x-4|} = \lim_{x \rightarrow 4^-} \frac{(x+4)(x-4)}{|x-4|} .$$

Since  $x$  approaches 4 from the left,  $x-4$  is negative,  $|x-4|$  is positive, and  $x+4$  is positive. So,  $\frac{(x+4)(x-4)}{|x-4|}$  is negative.

Because  $x-4$  is negative, we know that  $|x-4| = -(x-4)$ . This gives us:

$$\lim_{x \rightarrow 4^-} \left[ \frac{(x+4)\cancel{(x-4)}}{-\cancel{(x-4)}} \right] = \lim_{x \rightarrow 4^-} [-(x+4)]$$

$$= -8 .$$



$$\textcircled{9.} \text{ (c.) } \lim_{h \rightarrow 0} \frac{\sqrt{9+h} - 3}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\sqrt{9+h} - 3}{h} \left( \frac{\sqrt{9+h} + 3}{\sqrt{9+h} + 3} \right)$$

$$= \lim_{h \rightarrow 0} \frac{\cancel{(9+h)} - \cancel{3^2}}{h(\sqrt{9+h} + 3)} \leftarrow \text{(difference of two squares)}$$

$$= \lim_{h \rightarrow 0} \frac{\cancel{h}}{\cancel{h}(\sqrt{9+h} + 3)}$$

$$= \lim_{h \rightarrow 0} \frac{1}{\sqrt{9+h} + 3}$$

$$= \frac{1}{\sqrt{9+0} + 3} = \frac{1}{3+3} = \frac{1}{6} .$$