Sec 5 To Know - formulas + ideas + skills

$F = P\left(1 + \frac{r}{n}\right)^{nt}$ for n compounding periods at interest rate r over n years.

F = future value of loan or account (investment)

P = principal (present value) amt of loan or account (investment)

Compounded n times / year

$P = F\left(1 + \frac{r}{n}\right)^{-nt}$ to solve for P, simply move $\left(1 + \frac{r}{n}\right)^{nt}$ to other side by multiplying by $\left(1 + \frac{r}{n}\right)^{-nt}$ on both sides.

$F = Pe^{rt}$ for compounding "continuously" (that is, every minute of every day)

Even daily compounding ($n = 365$) is essentially continuous, so use this formula for "daily" rather than above with $n = 365$.

Compounded continuously

$P = Fe^{-rt}$ as before, solving for present value, multiply both sides by e^{-rt}.

Effective interest rate – what the rate is effectively as a result of compounding n times or continuously.

$$\text{eff int rate} = \left(1+\frac{r}{n}\right)^n - 1 \quad \text{or} \quad e^r - 1$$

That's because the first term of each formula here is the multiplier of the principal, in the final calc.

To isolate time t, use log props:

That is, if $a^t = b$ then $\ln a^t = \ln b$,

and so $t \ln a = \ln b$

(unknown; numerical values)

hence $$t = \frac{\ln b}{\ln a}$$

So $F = Pe^{rt}$ solved for t becomes:

$$\ln\left(\frac{F}{P}\right) = rt \ln e$$

$$\ln\left(\frac{F}{P}\right) = rt$$

$$t = \frac{\ln(F/P)}{r}$$

for continuous compounding.

Hence, "doubling" time, time needed to double investment P is as follows:

$$F = 2P = Pe^{rt}$$

$$2 = e^{rt}$$

$$\ln 2 = \ln e^{rt} = rt \ln e = rt \cdot 1$$

$$\ln 2 = rt$$

solve for t

$$t = \frac{\ln 2}{r}$$

ok to memorize

$$\frac{\ln 2}{r} = t$$

By the way, you can use any base, but when we cmpd. ctsly, base e is the obvious choice.

Questions?

$$2 = e^{rt}$$

$$\log_2 2 = \log_2 e^{rt}$$

$$1 = rt(\log_2 e)$$

just a number

$$\frac{1}{r \log_2 e} = t$$

$$t = \frac{\ln 2}{r}$$