Practice trig test — key

1a) $-150° + 360° = 210°$

$-150° - 360° = -550°$

b) $\frac{13\pi}{4} + 2\pi = \frac{21\pi}{4}$

$\frac{13\pi}{4} - 4\pi = -\frac{3\pi}{4}$

2a) $72°$ $\quad 90 - 72 = 18°$ comp

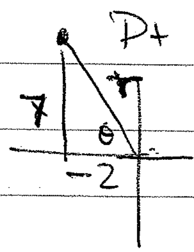
$180 - 72 = 108°$ supp

b) $\frac{\pi}{5}$ $\quad \frac{\pi}{2} - \frac{\pi}{5} = \frac{3\pi}{10}$ comp

$\pi - \frac{\pi}{5} = \frac{4\pi}{5}$ supp

3\. a) $210° \times \frac{\pi}{180°} = \frac{7\pi}{6}$

b) $5^r \times \frac{180°}{\pi^r} = \frac{900°}{\pi}$ (π is irrational so leave as is)

4\. 

$(-2, 7)$ is on terminal side of θ

$r = \sqrt{49+4} = \sqrt{53}$

$\sin\theta = \frac{7}{\sqrt{53}}$ $\qquad \csc\theta = \frac{\sqrt{53}}{7}$

$\tan\theta = \frac{-7}{2}$ $\qquad \cos\theta = \frac{-2}{\sqrt{53}}$ $\qquad \sec\theta = \frac{-\sqrt{53}}{2}$

$\cot\theta = \frac{-2}{7}$

b) $(\cos\theta, \sin\theta) = \left(\frac{-2}{r}, \frac{7}{r}\right) = \left(\frac{-2}{\sqrt{53}}, \frac{7}{\sqrt{53}}\right)$

5) Range $[-1, 3]$ $\longrightarrow$ Amp $= 4/2 = 2$

Period 6π $\longrightarrow$ $\frac{2\pi}{B} = 6\pi \rightarrow B = 1/3$

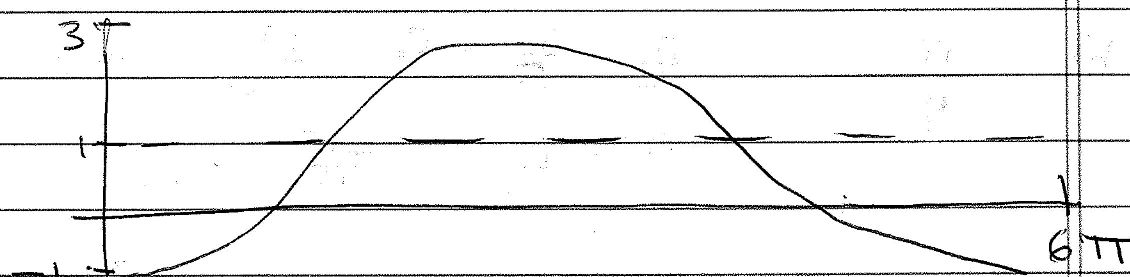
Reflects across x-axis $\longrightarrow$ Negative coeff.

a) $f(x) = -2\cos\left(\frac{x}{3}\right) + ?$

It is stretched vertically by factor of 2, but tops out at 3, so it must be lifted up 1.

$$f(x) = -2\cos\left(\frac{x}{3}\right) + 1$$

b)

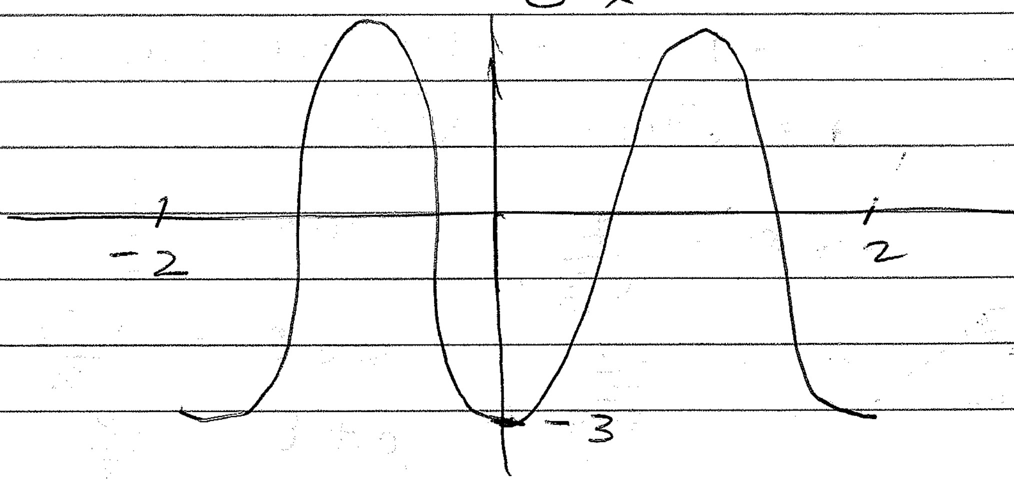


6) $f(x) = -3\cos(\pi x)$

$A = |-3| = 3$ $\qquad$ Per $= \frac{2\pi}{\pi} = 2$

Endpts: $0 = \pi x$ and $2\pi = \pi x$

$0 = x$ $\qquad$ $x = 2$



7) $\sin(7\pi) = 0$

$\cos\left(\frac{2\pi}{3}\right) = -1/2$

$\tan\left(\frac{3\pi}{4}\right) = -1$

$\sin\left(-\frac{\pi}{6}\right) = -\frac{1}{2}$

$$\cos\left(\frac{5\pi}{12}\right) = \cos\left(\frac{3\pi}{12} + \frac{2\pi}{12}\right) = \cos\left(\frac{\pi}{4} + \frac{\pi}{6}\right)$$

$$= \cos\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{6}\right) - \sin\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{6}\right)$$

$$= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \cdot \frac{1}{2}$$

$$= \frac{\sqrt{3} - 1}{2\sqrt{2}}$$

$$\tan(75^\circ) = \tan(45^\circ + 30^\circ)$$

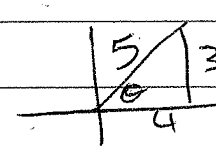
$$= \frac{\tan(45^\circ) + \tan(30^\circ)}{1 - \tan(45^\circ)\tan(30^\circ)}$$

$$= \frac{1 + 1/\sqrt{3}}{1 - 1(1/\sqrt{3})} = \frac{\sqrt{3} + 1}{\sqrt{3} - 1}$$

8. $\arccos(-1) = \pi$

$\tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$

$\cos\left(\tan^{-1}(3/4)\right) = \cos(\theta)$ such that $\tan\theta = \frac{3}{4}$



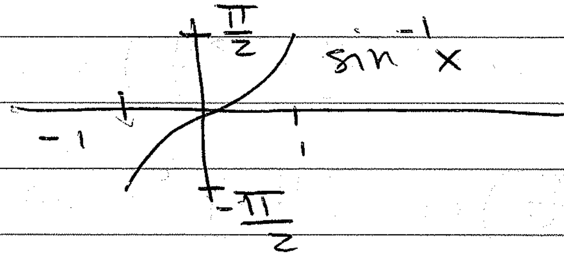
$\cos\theta = \frac{3}{5}$

So far, the domain was not an issue.

But watch out now:

$$\arcsin\left(\sin\left(\frac{5\pi}{3}\right)\right)$$

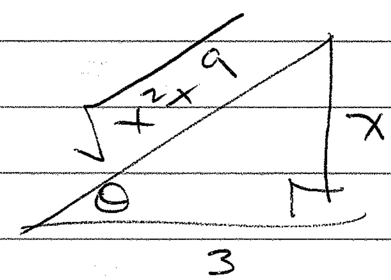
Need ref $\measuredangle$ for $\frac{5\pi}{3}$ in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, but consider the fact that $\sin\left(\frac{5\pi}{3}\right) < 0$



$$\arcsin\Big(\underbrace{\sin \frac{5\pi}{3}}_{\text{ratio}}\Big)$$

$$\arcsin\left(-\sqrt{3}/2\right) = -\frac{\pi}{3}$$

$$\sec\Big(\underbrace{\arctan(x/3)}_{\theta}\Big)$$



$$\sec\theta = \frac{\sqrt{x^2+9}}{3}$$

9. $\tan x + \dfrac{\cos x}{1+\sin x} \stackrel{?}{=} \dfrac{1}{\cos x}$

$\dfrac{\sin x}{\cos x} + \dfrac{\cos x}{1+\sin x} \stackrel{?}{=}$

$\dfrac{\sin x\,(1+\sin x) + (\cos x)(\cos x)}{\cos x\,(1+\sin x)} = ?$

$\dfrac{\sin x + \sin^2 x + \cos^2 x}{(\cos x)(1+\sin x)} =$

$\dfrac{\cancel{(\sin x + 1)}}{(\cos x)\cancel{(1+\sin x)}} = \dfrac{1}{\cos x}$ ✓

$\dfrac{\sin(\alpha+\beta)}{\sin(\alpha)\sin(\beta)} \stackrel{?}{=} \cot\alpha + \cot\beta$

$\dfrac{\sin\alpha\cos\beta + \cos\alpha\sin\beta}{\sin\alpha\sin\beta} = ?$

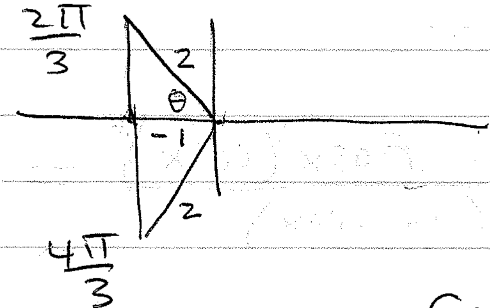
$\cancel{\dfrac{\sin\alpha\cos\beta}{\sin\alpha\sin\beta} + \dfrac{\cos\alpha\sin\beta}{\sin\alpha\sin\beta} \neq}$?

$\dfrac{\cancel{\sin\alpha}\cos\beta}{\cancel{\sin\alpha}\sin\beta} + \dfrac{\cos\alpha\cancel{\sin\beta}}{\sin\alpha\cancel{\sin\beta}} = \cot\beta + \cot\alpha$ ✓

10. a) $2\cos(3x) = -1$

$$\cos(3x) = -\frac{1}{2} \qquad \left(\text{Per} = \frac{2\pi}{3}\right)$$

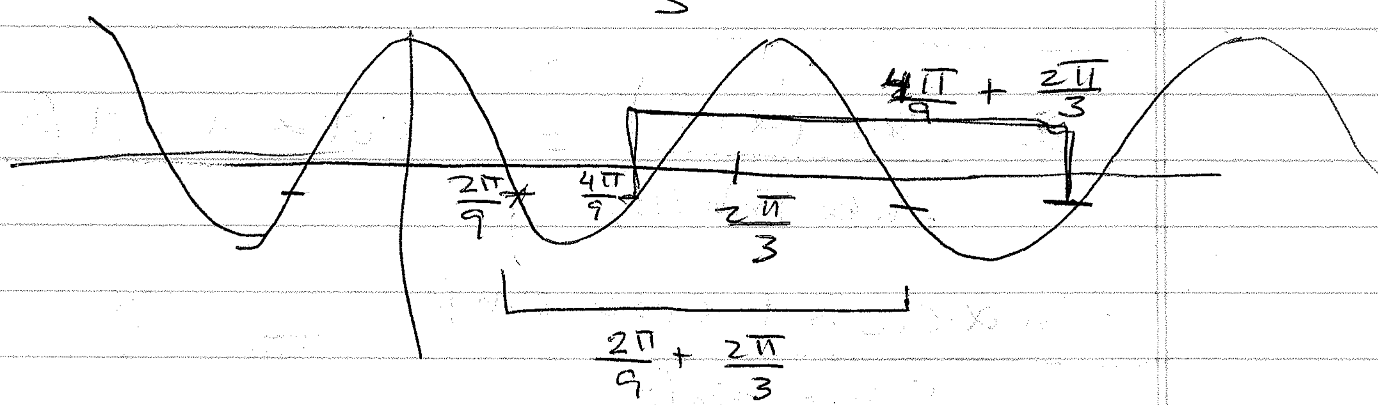
that is, $\cos\theta = -\frac{1}{2}$



$$\theta = \frac{2\pi}{3} = 3x \longrightarrow x = \frac{2\pi}{9}$$

$$\text{and } \theta = \frac{4\pi}{3} = 3x \longrightarrow x = \frac{4\pi}{9}$$

The fcn. is periodic + the solus. are infinite, but not every 2π, but every $\frac{2\pi}{3}$,



So the set of answers is

$$x = \frac{2\pi}{9} + \frac{2\pi}{3}n$$

$$\text{and } \frac{4\pi}{9} + \frac{2\pi}{3}n$$

10 b) Find the solns. on $[0, 2\pi]$

$$1 + \cos x = 2\sin^2 x$$

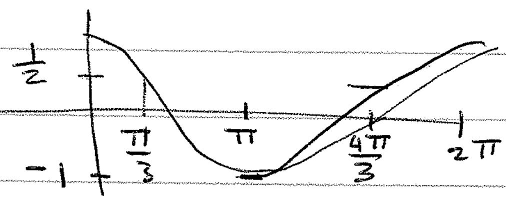
$$1 + \cos x = 2(1 - \cos^2 x)$$

$$1 + \cos x - 2 + 2\cos^2 x = 0$$

$$2\cos^2 x + \cos x - 2 = 0$$

$$\underbrace{(2\cos x - 1)}_{\cos x = \frac{1}{2}}\underbrace{(\cos x + 1)}_{\cos x = -1} = 0$$

$$\boxed{x = \frac{\pi}{3}, \frac{4\pi}{3}}$$



$$\boxed{x = \pi}$$

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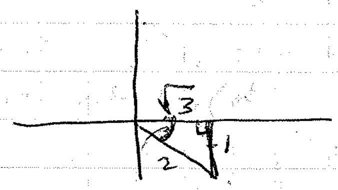
↗ i.e. #11

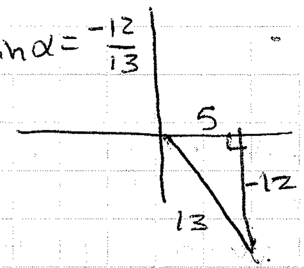
#14) Suppose $\cos\alpha = 5/13$, $\frac{3\pi}{2} < \alpha < 2\pi$

$\sin\beta = -\frac{1}{2}$, $\frac{3\pi}{2} < \beta < 2\pi$

a) $\cos\beta = \frac{\sqrt{3}}{2}$



b) $\sin\alpha = \frac{-12}{13}$



c) $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$$= -\frac{12}{13}\cdot\frac{\sqrt{3}}{2} + \frac{5}{13}\cdot\frac{-1}{2}$$

$$= \frac{-12\sqrt{3}}{26} - \frac{5}{26} = \frac{-12\sqrt{3}-5}{26}$$

d) What quadrant is $\alpha+\beta$ in?

$$\begin{array}{rl} & \frac{3\pi}{2} < \alpha < 2\pi \\ + & \frac{3\pi}{2} < \beta < 2\pi \\ \hline & 3\pi < \alpha+\beta < 4\pi \end{array}$$

that is, either QIII or IV

~~d~~e) $\cos(2\alpha) = \cos^2(\alpha) - \sin^2(\alpha)$

$$= \left(\frac{5}{13}\right)^2 - \left(-\frac{12}{13}\right)^2 = \frac{25 - 144}{169} = \frac{-119}{169}$$

f) What quadrant is 2α in?

Again $\frac{3\pi}{2} < \alpha < 2\pi \xrightarrow{\text{so} \otimes 2} 2\left(\frac{3\pi}{2}\right) < 2\alpha < 2(2\pi)$

or $3\pi < 2\alpha < 4\pi$

g) $\sin\left(\frac{\beta}{2}\right) = \pm\sqrt{\frac{1-\cos\beta}{2}} = \pm\sqrt{\frac{1-\sqrt{3}/2}{2}}$ ok to leave unsimplified

$= \pm\sqrt{\frac{2-\sqrt{3}}{4}}$ using LCD of 2

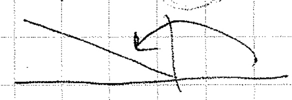
But, which answer is it? Positive or negative? See (h), then finalize this.

h) What quadrant is $\beta/2$ in?

Again, go to domain $\frac{3\pi}{2} < \beta < 2\pi$

Divide by 2: $\frac{\frac{3\pi}{2}}{2} < \frac{\beta}{2} < \frac{2\pi}{2}$

$\frac{\beta}{2}$ is in Q II $\longleftarrow$ $\frac{3\pi}{4} < \frac{\beta}{2} < \pi$



So $\sin\left(\frac{\beta}{2}\right) = +\sqrt{\frac{2-\sqrt{3}}{4}}$