

# Key

## Math 108 Sections 1 and 4 Practice Test 1

1. Change  $1.\overline{2090909\dots}$  to a fraction (hint: let  $x = 1.2090909\dots$ , etc.)  
 Because there are two repeating places, we multiply both sides of  $x = 1.2090909\dots$  by 100 (2 zeroes), then subtract.

$$100x = \cancel{120.909090\dots}$$

$$-x = \cancel{1.209090\dots}$$

$$99x = 119.7$$

$$x = \frac{119.7}{99} = \boxed{\frac{1197}{990}}$$

2. True or false:

$1 - \sqrt{7}$  is a rational number false

$3.55555\dots$  is rational number. true

3. Simplify the following expression, leaving no negative exponents:

$$\left(\frac{xy^5}{3x^2yz}\right)^{-2} = \left(\frac{3x^{-2}y^2z^2}{xy^5}\right)^2 = \frac{3^2 x^{-4} y^2 z^2}{x^2 y^{10}} = \frac{9 y^2 z^2}{x^2 x^4 y^{10}} = \boxed{\frac{9 z^2}{x^6 y^8}}$$

$$\left(\frac{a+b}{b^2+a}\right)^{-1} = \left(\frac{a^2+b^3}{ab^2}\right)^{-1} = \boxed{\frac{ab^2}{a^2+b^3}}$$

$$\text{LCD} = ab^2$$

4. Simplify the following radical expression as far as possible:

$$\sqrt{250} - \sqrt[3]{1000} + 2\sqrt[3]{81} - \sqrt[3]{24} = \sqrt{25 \cdot 10} - 10 + 2\sqrt[3]{27 \cdot 3} - \sqrt[3]{8 \cdot 3} \\ = 5\sqrt{10} - 10 + 2 \cdot 3\sqrt[3]{3} - 2\sqrt[3]{3} = \boxed{5\sqrt{10} - 10 + 4\sqrt[3]{3}}$$

Simplify  $\overbrace{(2^4)}^{\text{Simplify}} = \boxed{-16}$   
 Change to radical form  $= \boxed{-2^4}$

$$= (32^{3/5})^3 = \boxed{8}$$

$$-x^{3/2} = -(\overbrace{x^{3/2}}^{\text{Change to radical form}}) = \boxed{-\sqrt{x^3}} \text{ or } -(\sqrt{x})^3$$

5. Rationalize the denominator:  $\frac{3}{\sqrt{2}+3} \cdot \frac{\sqrt{2}-3}{\sqrt{2}-3} = \frac{3\sqrt{2}-9}{4-9} = \boxed{\frac{3\sqrt{2}-9}{-5}}$

6. For the function  $f(x) = x^2 + 3$ , find the difference quotient  $\frac{f(x+h)-f(x)}{h}$ .

$$f(x+h) = (x+h)^2 + 3 = x^2 + 2xh + h^2 + 3$$

$$f(x) = x^2 + 3$$

$$\frac{f(x+h) - f(x)}{h} = \frac{x^2 + 2xh + h^2 + 3 - x^2 - 3}{h}$$

$$= \frac{2xh + h^2}{h} = \boxed{2x + h}$$

7. Write the domains of the following in interval notation

$$f(x) = x^2 - 3x + 5 \quad \boxed{(-\infty, \infty)} \quad \mathbb{R}$$

$$f(x) = \frac{x+2}{3x-5} \quad 3x-5 \neq 0 \quad \text{so} \quad x \neq \frac{5}{3}$$

$$\boxed{(-\infty, \frac{5}{3}) \cup (\frac{5}{3}, \infty)}$$

$$g(x) = \frac{1}{\sqrt{2x-3}} \quad 2x-3 > 0 \quad \text{so} \quad x > \frac{3}{2}$$

$$\boxed{(\frac{3}{2}, \infty)}$$

7. Factor completely

$$30x^4y^2z + 12xy^3z^4 - 6x^2yz^2 = \boxed{6xyz(5x^3y + 2y^2z^3 - xz)}$$

$$x^6 - 1 = \boxed{(x^3 + 1)(x^3 - 1) = (x+1)(x^2 - x + 1)(x-1)(x^2 + x + 1)}$$

$-3x^2 - 5x - 6$  = not factorable! (I'm sorry. Sign + typo.)

$$4x^3 - 12x^2 - 9x + 27 = \underbrace{\quad}_{\quad} \cdot 4x^2(x-3) - 9(8x-3) = (x-3)(4x^2-9)$$

$$= \boxed{(x-3)(2x-3)(2x+3)}$$

8. Solve for  $x$  by completing the square:

$$-3x^2 + 12x - 5 = 0 \quad (\text{solve by completing the square})$$

$$-(3x^2 - 12x + 5) = 0$$

$$-3(x^2 - 4x) + 5 = 0$$

$$-3(x^2 - 4x + (-\frac{4}{2})^2) = -5 + ?$$

$$\sqrt{x-9} - \sqrt{x} = -1 \quad (\text{check your answers!!!!})$$

$$\begin{aligned} -3(x-2)^2 &= -5 + 12 \\ (x-2)^2 &= +17 \\ x-2 &= \pm \sqrt{\frac{17}{3}} \end{aligned}$$

$$\boxed{x = 2 \pm \sqrt{\frac{17}{3}}}$$

$$(\sqrt{x-9} - \sqrt{x})^2 = (-1)^2$$

$$x-9 - 2\sqrt{x}\sqrt{x-9} + x = 1$$

$$x-9 + x - 1 = 2\sqrt{x}\sqrt{x-9}$$

$$2x-10 = 2\sqrt{x}\sqrt{x-9}$$

$$x-5 = \sqrt{x}\sqrt{x-9}$$

$$(x-5)^2 = x(x-9)$$

$$\begin{aligned} x^2 - 10x + 25 &= x^2 - 9x \\ 25 &= x \end{aligned}$$

9. Perform the following function composition where  $f(x) = \sqrt{x+1}$   $g(x) = \frac{2}{x^2}$ :

$$f \circ g = f\left(\frac{2}{x^2}\right) = \sqrt{\frac{2}{x^2} + 1}$$

$$g \circ f = g(\sqrt{x+1}) = \frac{2}{\sqrt{x+1}^2} = \frac{2}{x+1}$$

What is the domain of  $f \circ g$ ? (Hint: First name the domain of each function.)

$$\text{Dom } g : x \neq 0$$

$$\boxed{\text{Dom } f \circ g : x \neq 0}$$

What is the domain of  $g \circ f$ ?

$$\text{Dom } f : x \geq -1$$

$$\boxed{\text{Dom } g \circ f : x > -1}$$

will not notice the radicand ~~can't~~ be negative, even with  $x < 0$ .

For  $f \circ g$ , find  $\text{dom } g$ , then  $\text{dom } f(g)$  as it is affected by  $\text{dom } g$ .  
Do opposite for  $g \circ f$ .

10. Find the inverse of the function  $f(x) = 6 - x^2$  on  $[0, \infty)$

~~you don't graph on this test; showing 1-1 half of  $f(x)$~~

$$y = 6 - x^2$$

$$x = 6 - y^2$$

$$y^2 = 6 - x$$

$$y = \pm \sqrt{6-x} = f^{-1}(x)$$

Since  $f^{-1}$  starts at  $\sqrt{6-6} = 0$  and goes to  $\infty$ .

11. Show by definition whether the functions are odd, even or neither.

$$f(x) = x^2 - 5$$

$$f(-x) = (-x)^2 - 5 = x^2 - 5 = f(x) \quad \boxed{\text{even}}$$

$$f(x) = 10x + 5$$

$$f(-x) = 10(-x) + 5 = -10x + 5 \neq -(10x + 5) = -f(x)$$

$$f(x) = 2(x+1)^3$$

$$f(-x) = 2(-x+1)^3 = 2(-1(x-1))^3 = 2(-1)^3(x-1)^3 = -2(x-1)^3 \neq f(x) \text{ or } f(-x)$$

12. Evaluate the sum

$$\sum_{i=40}^{200} i = \sum_{i=1}^{200} i - \sum_{i=1}^{39} i = \left[ \frac{200(201)}{2} - \frac{39(40)}{2} \right] \quad \begin{matrix} \text{Gauss} \\ \text{formula} \end{matrix}$$

$$\sum_{i=1}^{15} (7i^2 - 4i)$$

$$= 7 \sum_{i=1}^{15} i^2 - 4 \sum_{i=1}^{15} i$$

The formula

for  $\sum_{i=1}^n i^2$

Will be given

Gauss

$$= 7 \left( \frac{15(15+1)(2 \cdot 15 + 1)}{6} \right) - 4 \left( \frac{15(16)}{2} \right)$$

given

Rewrite the sum as its equivalent sum having 11 as an upper bound.

Given  $\sum_{i=4}^{13} \frac{i+2}{i^2} = \sum_{i=?}^{11} ?$

This is 13-2  
so we need  
to subtract 2  
from the lower  
bound:  $4-2=2$

$$= \sum_{i=2}^{11} ?$$

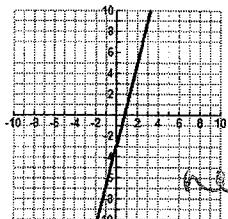
Since we decreased the counter by 2, we have to add 2 to i in the  $a_i$  term to make up for it

$$= \boxed{\sum_{i=2}^{11} \frac{(i+2)+2}{(i+2)^2}}$$

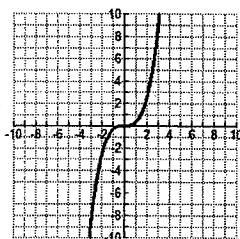
## Even, Odd, or Neither Worksheet

Determine whether the following functions are even, odd, or neither by the look of the graphs. Odd are symmetric over the origin, that is,  $(x,y)$  is reflected to  $(-x, -y)$ . Even are symmetric over the  $y$ -axis, that is  $(x,y)$  is reflected to  $(-x,y)$ . *Odd funcs must have a point at  $(0,0)$*   
*if zero is in the domain.*

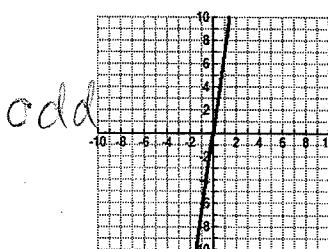
1.  $f(x) = 4x - 3$



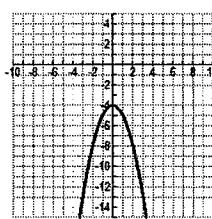
4.  $f(x) = \frac{1}{3}x^3$



5.  $f(x) = 7x$

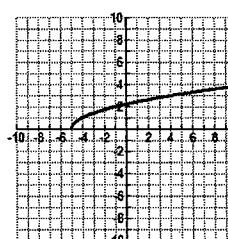


3.  $f(x) = -x^2 - 4$



Even

6.  $f(x) = \sqrt{x+5}$



Neither

**Use the definitions to decide if the following are odd, even or neither. Start by finding  $f(-x)$  and seeing if it equals  $f(x)$  or  $-f(x)$ .**

7.  $f(x) = 3x^2$

$$f(-x) = 3(-x)^2$$

$$= 3x^2$$

$$= f(x) : \text{even}$$

8.  $f(x) = x^3 - 2$

$$f(-x) = (-x)^3 - 2$$

$$= -x^3 - 2$$

$$f(x) \neq -x^3 + 2 \quad \text{not even}$$

neither

9.  $f(x) = 3x + 4$

$$f(-x) = -3x + 4$$

$$\neq f(x)$$

$$\text{nor } -f(x)$$

10.  $f(x) = x^2 - 5$

$$f(-x) = (-x)^2 - 5$$

$$= x^2 - 5$$

$$= f(x)$$

even

11.  $f(x) = 10x + 5$

$$f(-x) = -10x + 5$$

neither

12.  $f(x) = 2(x+1)^3$

$$f(-x) = 2(-x+1)^3$$

$$= 2(1-x)^3$$

$\neq f(x)$  nor  
 $-f(x)$

neither

