Review Problems for MATH 220 – Exam 3 (Sections 17-21, 22-24, 27, 29)

Partial derivatives / Lagrange Multipliers Method

Find all first and second partials for $g(x, y) = 2y \ln x + e^y$

Find all first and second partials for $h(x, y) = 6xy^2 - 4x^3y$

Find all first and second partials for $f(x, y) = 2x^2 + 4xy^3 - 6y^2$

Find all first and second partials for $f(x, y) = (3x^2 - 5xy)(4 \ln x)$

Find all first and second partials for $f(x, y) = (6e^y)(3xy^4 - 3x^2y^2)$

Find all first and second partials for $f(x, y) = \frac{3x^2y^2 - 6x^3}{5xy + 4y}$

Find all first and second partials for $f(x, y) = \ln(5xy + 3x^2 - 2y^4)$

A shipping company handles rectangular boxes provided the sum of the length, width, and height does not exceed 96 inches. Find the dimensions of the box that meets this condition and has the largest volume.

A company has a monthly advertising budget of $60,000. Their marketing department estimates that if they spend x dollars on newspaper advertising and y dollars on television advertising, then the monthly sales will be given by $S = 90x^{\frac{1}{4}}y^{\frac{3}{4}}$ dollars. Determine how much money the company should spend on newspaper ads and on television ads per month to maximize its monthly sales.

Find the largest product the positive numbers x, y, and z can have if $x + y + z^2 = 16$.

A satellite in the shape of the ellipsoid $4x^2 + y^2 + 4z^2 = 16$ enters Earth's atmosphere and its surface begins to heat. After one hour, the temperature at the point (x, y, z) on the satellite's surface is given by $T(x, y, z) = 8x^2 + 4yz - 16z + 600$. Find the hottest point on the satellite's surface.

A company manufactures two types of athletic shoes: jogging shoes and cross-trainers. The total revenue from x units of jogging shoes and y units of cross trainers is given by $R(x, y) = -5x^2 - 8y^2 - 2xy + 42x + 102y$, where x and y are in thousands of units. Find the values of x and y to maximize the total revenue.

The United States Postal Service declares a box to be oversized if its height plus its girth (perimeter of its base) is more than 108 inches. Find the largest volume a box can have and not be considered oversized.

Absolute Extrema / Old school optimization

The population in a small town from the beginning of the year 2000 to the beginning of the year 2010 is modeled by the function $P(t) = t^3 - 15t^2 + 63t + 10{,}000$, where $P(t)$ represents the population in the town t years after the start of 2000 (in other words, $t = 0$ is the start of the year 2000). Answer the following questions.

a) On what time interval(s) was the population in the town declining? Justify.

b) Find the maximum population in the town over the 10-year interval.

Given $f(x) = \frac{4x}{x^2+9}$, find the absolute maximum and minimum on $[0, 5]$.

Given $f(x) = \frac{1-x}{x^2+3x}$, find the absolute extrema on $[1, 4]$.

Given $f(x) = 2x^3 - 6x^2 - 48x + 7$, find the absolute maximum and minimum on $[-6, 8]$.

The cost to build q items is $6000 + 5q + 0.01q^2$. In order to sell q items, the price will need to be $p(q) = 20 - \frac{q}{4}$. Find the quantity that will maximize profit.

A rectangular enclosure is to be built next to a river. There will be no fence on the river side of the enclosure. The cost for the material for the side paralle to the river is $6.00 per foot. The material for the sides perpendicular to the river costs $2.00 per foot. There is a budget of $240.00 for the fence. What dimensions of the fence result in the largest enclosed area?

A movie theater has a seating capacity of 525 people. With the ticket price at $10, average attendance at a movie has been 375 people. Management has decided to lower admission prices to boost attendance. A market survey indicates that for each two dollars the price of a ticket is lowered, average attendance will increase by 100. What ticket price will maximize revenue from ticket sales?

A 12 foot piece of wire is cut into two pieces. The first piece is bent into a square and the second piece is bent into an equilateral triangle. Where should the wire be cut to minimize the combined area of the two resulting shapes?

A Norman window is in the form of a rectangle surmounted by a semicircle. Find the dimensions of the window that will admit the most light.

Curve sketching

Suppose $f(x) = 3x^3 - 9x + 7$. Graph.

Suppose $g(x) = \frac{x^2}{3x-2}$. If so, $g'(x) = \frac{3x^2-4x}{(3x-2)^2}$ and $g''(x) = \frac{8}{(3x-2)^3}$. Graph.

Suppose $f(x) = \frac{x^2-1}{x^3}$. If so, $f'(x) = \frac{(3-x^2)}{x^4}$ and $f''(x) = \frac{2(x^2-6)}{x^5}$. Graph.

Suppose $h(x) = x^{\frac{2}{3}}\left(\frac{5}{2} - x\right)$, so $h'(x) = \frac{5(1-x)}{3x^{\frac{1}{3}}}$ and $h''(x) = \frac{-5(1+2x)}{9x^{\frac{4}{3}}}$. Graph.

Suppose $f(x) = \frac{2x^2-6x}{3x^2-8x-3}$. Find the equations of all horizontal/vertical asymptotes.

Suppose $f(x) = 2x(x-4)^3$. Find all local extrema. Classify each as a local maximum or local minimum.

Elasticity of demand

The demand function for a particular product is $q = \sqrt{50 - p^2}$. At a price of \$3.00, is demand elastic or inelastic? If price is increased slightly, does revenue go up or down?

Suppose $3p + \sqrt{q} = 800$ indicates the relationship between price p and demand q for some commodity.

a) Find $E(p)$.

b) Does the commodity in this example exhibit elastic or inelastic demand at $p = \$70$. Justify your answer using the result from part (a).

c) Hence, to increase revenue, should the company raise or lower the price?

d) Use $E(p)$ to determine the price that should be charged to maximize revenue. Give your answer to the nearest dollar.

Suppose the demand for a product is given by $q = 1500 - 0.05p^2 - 0.2p$.

a) Find $E(p)$.

b) If the price is \$100/item, would a slight increase in price lead to an increase or decrease in revenue?

c) Find the price that maximizes revenue using $E(p)$.