MATH 220 – Review Problems (Exam 2)

DERIVATIVES WITH SHORTCUTS

$$f(x) = 4x^9 + 7x^5 - 22x^2 + 17 + \frac{2}{x^3} + 4^x$$

$$f(x) = \frac{x^3 + 9x + 2}{10x^2 - 6x + 3}$$

$$f(x) = \left(\frac{2x^3 - 1}{5x^4 - 9x}\right)^3$$

$$f(x) = \frac{9}{(x^3 + 2x + 7)^8}$$

$$f(x) = e^{4x}\ln(x^2 + 9)$$

$$f(x) = 2^{3x+1}$$

$$f(x) = \log_3(30x^2 + 9x - 5)$$

$$f(x) = \sqrt{\mathrm{x}^3 \ln x}$$

$$f(x) = \sqrt[5]{x^2} + \sqrt[3]{x^7} + \frac{1}{x^3} + e$$

$$f(x) = e^{\frac{5x+1}{2x-3}}$$

$$f(x) = \frac{(4x^3 - 5x^2 + 2)^3}{1 + e^{2x}}$$

Suppose f and g are both differentiable functions for their entire domains. Function values and values of their derivatives when $x = 5$ and $x = -3$ are given in the following table:

x	f	g	f'	g'
5	2	-3	4	6
-3	8	0	-2	1

Using the above information, determine the following values, if they exist. If a value does not exist, explain why. Show your work.

a) $(f + g)'(-3)$ b) $(fg)'(5)$ c) $\left(\frac{f}{g}\right)'(-3)$ d) $(f \circ g)'(5)$

HIGHER DERIVATIVES

Suppose $f(x) = \frac{x^3}{1+x^2}$. Find $f'(x)$ and $f''(x)$.

Suppose $f(x) = e^{5x^3}$. Find $f'(x)$ and $f''(x)$.

Suppose $f(x) = 10x^6 + e^{7x}$. Find $f'(x)$ and $f^{(n)}(x)$.

IMPLICIT DIFFERENTIATION

Find $\frac{dy}{dx}$ if $x^2y^2 + x \ln y = 4$.

Find $\frac{dy}{dx}$ if $1 + x + y^3 = e^{xy^2}$.

Suppose $x^2 + 2xy - y^2 + x = 2$. Find the equation of the line tangent to this curve at $(1,2\ )$.

Suppose $y + x\sqrt{y} = 8$. Find the tangent line at (2, 4).

Suppose $x^2 + 2x + y^3 = 35$. Find the tangent line when $x = 2$.

RELATED RATES

Gravel is being dumped from a conveyor belt at a rate of 30 cubic feet per minute such that it forms a pile in the shape of a cone whose base diameter and height are always equal. How fast is the height of the pile increasing when the pile is 10 feet high?

At noon, ship A is 150 km west of ship B. Ship A is sailing east at 35 km/h and ship B is sailing north at 25 km/h. How fast is the distance between the ships changing at 4:00 PM?

A spotlight on the ground shines on a wall 12 meters away. If a man 2 meters tall walks from the spotlight toward the wall at a speed of 1.6 meters per second, how fast is the length of his shadow on the building decreasing when he is 4 meters from the building?

Suppose a triangle has a constant area of $30\ \text{cm}^2$, and that the base of the triangle is increasing at a rate of 2 centimeters per second. How fast is the height of the triangle changing when the base is 10 centimeters long?

A balloon is rising at a constant rate of 5 feet per second. A boy is bicycling along a straight road at a speed of 15 feet per second. When he passes under the balloon, it is 45 feet above him. How fast is the distance between the boy and the balloon increasing 3 seconds later?

CRITICAL NUMBERS

Find the critical numbers for:

$f(x) = 8x^3 - 12x^2 - 48x$

$f(x) = x^{\frac{3}{5}}(4 - x)$

$f(x) = x\sqrt{4 - x^2}$

$f(x) = \frac{x^2 - 5x + 4}{x - 2}$

$f(x) = x - 3x^{\frac{1}{3}}$

$f(x) = \frac{4x}{x^2 + 1}$

$f(x) = \frac{x^2 + 5x + 1}{x^2}$

MARGINALS

A business determines that the relationship between the price of their product and the quantity sold is given by $q = 64 - (p - 5)^3$ for $0 \leq p \leq 14$, where q is the quantity and p is the price (in dollars).

a) Determine the revenue function $R(p)$, which gives the revenue the business will make, in dollars, when the product is sold for p dollars.

b) Find $\frac{dR}{dp}$ when $p = 7$. Discuss the real-world significance of this value. In particular, at this price, what happens to the revenue as the price increases?

c) Determine the revenue function $R(q)$, which gives the revenue the business will make, in dollars, when q units of the product are sold.

d) How much revenue does the company make when 72 units are sold?

e) When 72 units are sold, is revenue increasing or decreasing? Fully justify your answer.

Suppose the relationship between the demand q for a product and its price p is given by the relationship $q = 100\sqrt{25 - p}$, for $0 \leq p \leq 25$. Suppose the cost to produce the product is given by $C(q) = 30q + 1200$.

a) Find the marginal cost. Explain the meaning of your answer.

b) Find an expression for revenue as a function of the price of the product.

c) Find dR/dp when $p = \$9$. Explain the meaning of your answer.

d) Find an expression for profit as a function of the quantity produced and sold.

e) Find dP/dq. Explain why your answer indicates that the business is struggling.

Suppose $p = \frac{1000}{\sqrt{q}}$ where q is the demand for a particular product at price p dollars.

Suppose the cost to produce q units is given by $C(q) = 10q + 100\sqrt{q} + 10{,}000$

Determine the marginal cost, marginal revenue, and marginal profit at $q = 100$ units.