Practice for test 1

1a) $\lim_{x\to 4}\left(\sqrt{x}+\frac{3x}{x+2}+x^2+\log_4 x\right) = 2+\frac{12}{6}+4^2+\log_4 4$

$= 2+2+16+1 = 21$ //

b) $\lim_{x\to 3}\frac{2x(x-3)}{(x+6)(x-3)} = \frac{6}{9} = \frac{2}{3}$ //

c) $\lim_{x\to 4}\left(\frac{\sqrt{x}-2}{x-4}\,\frac{\sqrt{x}+2}{\sqrt{x}+2}\right) = \lim_{x\to 4}\frac{(x-4)\,1}{(x-4)(\sqrt{x}+2)} = \frac{1}{6}$ //

d) $\lim_{x\to -2^-}\frac{x+1}{(x+2)^2} = \frac{-2+1}{0} = \frac{-1}{0} = ?$

As $x\to -2^-$, $(x+2)^2 \longrightarrow 0$ in the positive sense

so $\frac{-1}{(x+2)^2} \longrightarrow -\infty$ (e.g. $(-2.1+2)^2 = (-.1)^2 = .01$, $(-2.01+2)^2 = (-.01)^2 = .0001$)

2.a) $f'(x) = \lim_{h\to 0}\frac{5(x+h)^2-8(x+h)+1-(5x^2-8x+1)}{h}$

$= \lim_{h\to 0}\frac{5(x^2+2xh+h^2)-8x-8h+1-5x^2+8x-1}{h}$

$= \lim_{h\to 0}\frac{5x^2+10xh+5h^2-8x-8h+1-5x^2+8x-1}{h}$

$= \lim_{h\to 0}\frac{10xh+5h^2-8h}{h} = 10x+5(0^2)-8 = 10x-8$ //

b) $y - y_1 = m(x - x_1)$

$y - f(x_1) = f'(x_1)(x - x_1)$

$y - f(1) = f'(1)(x - 1)$

$y - (-2) = f'(1)(x - 1)$

$f'(1) = 10(1) - 8 = 2$ from part (a).

so $\boxed{y + 2 = 2(x - 1)}$ is eqn. //

where $x_1 = 1$ (given)

$f(1) = 5 - 8 + 1$

$= -2$

3. $V(t) = 4\sqrt{t}$

a) Avg rate of change of V $= \frac{\Delta V}{\Delta t} = \frac{V(9) - V(1)}{9 - 1} = \frac{12 - 4}{8} = 1$

$\boxed{\$1/\text{mo}}$

b) $V_{\text{inst rate of change at time } t} = V'(t) = \lim_{h \to 0} \frac{4\sqrt{t+h} - 4\sqrt{t}}{h}$

$= 4 \lim_{h \to 0} \frac{\sqrt{t+h} - \sqrt{t}}{h} \cdot \frac{\sqrt{t+h} + \sqrt{t}}{\sqrt{t+h} + \sqrt{t}}$

$= 4 \lim_{h \to 0} \frac{t + h - t}{h(\sqrt{t+h} + \sqrt{t})} = 4 \cdot \frac{1}{\sqrt{t+0} + \sqrt{t}} = \frac{4}{2\sqrt{t}} = \frac{2}{\sqrt{t}}$

$V'(t) = \frac{2}{\sqrt{t}}$, $V'(4) = \frac{2}{\sqrt{4}} =$ **\$1/mo**

IROC at $t=4$ mos.

4. a) True

b) False

c) Not written! Here's one: T or F?

"A fcn is cts. at $x=a$ if $\lim_{x\to a^-} f = \lim_{x\to a^+} f$" **False**

(That is, if limit of f exists at $x=a$).

We also need that $a \in$ Dom f

and that $\lim_{x\to a} f(x) = f(a)$