3. $C(q) = 6000 + 5q + 0.01q^2$

$p(q) = 20 - \frac{q}{4}$, $R(q) = p \cdot q$

$= \left(20 - \frac{q}{4}\right)q$

$P(q) = R(q) - C(q)$

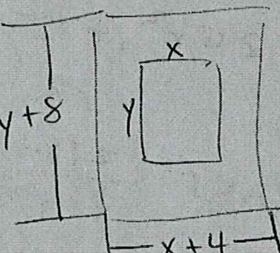
$= \left(20q - \frac{q^2}{4}\right) - \left(6000 + 5q + 0.01q^2\right)$

$P'(q) = 20 - \frac{2q}{4} - 5 - 0.02q$

$= 20 - .5q - 5 - 0.02q = 15 - .52q = 0$

So $q = \frac{15}{.52} \approx 29$ items.

4.



Let print area of $648\ cm^2 = xy$
Adding margins, 4 cm each on top + bottom gives $y+8$ and 2 cm each sides gives $x+4$

Minimize $A_{paper} = (y+8)(x+4)$, where $xy = 648$ or $y = \frac{648}{x}$

Then $A(x) = \left(\frac{648}{x} + 8\right)(x+4) = 648 + \frac{(648)(4)}{x} + 8x + 32$

$A'(x) = -\frac{(648)(4)}{x^2} + 8 = 0 \longrightarrow 8x^2 = (648)(4)$

$x^2 = 324$, $\boxed{\begin{array}{l} x = 18\text{cm} \\ y = 36\text{cm} \end{array}}$

5. $SA_{cost} = 3\pi r^2 + 2 \cdot 2\pi r h$, $V = \pi r^2 h$

$h = \frac{V}{\pi r^2}$

$SA_{cost} = 3\pi r^2 + 4\pi r \frac{V}{\pi r^2}$

$SA_{cost} = 3\pi r^2 + \frac{4V}{r}$

$SA'_{cost} = 6\pi r - \frac{4V}{r^2} = \frac{6\pi r^3 - 4V}{r^2} =$

$6\pi r^3 = 4V$

$6\pi r^{\cancel{3}} = 4\cancel{\pi}\cancel{r^2}h \longrightarrow 6r = 4h$

or $r = \frac{4}{6}h = \frac{2}{3}h$

$\boxed{r = 2/3\, h}$

$h = \frac{6.89}{\pi r^2}$

6. $V = 6.89 \text{ in}^3 = \pi r^2 h \longrightarrow$

$SA = 2\pi r^2 + 2\pi r h = 2\pi r^2 + \frac{2\cancel{\pi} \cancel{r} \cdot 6.89}{\cancel{\pi} r^{\cancel{2}}}$

$\frac{2(6.89)}{r}$

$SA' = 4\pi r - \frac{13.78}{r^2} = 0$

$4\pi r^3 - 13.78 = 0$

$r^3 = \frac{13.78}{4\pi}$,

$r = \sqrt[3]{\frac{13.78}{4\pi}}$

$h = \frac{6.89}{\pi \sqrt[3]{\frac{13.78}{4\pi}}^{\,2}}$

Use calculator + compare to actual can of soda

7. $R(n) = p(n) \cdot q(n)$

where n = # of \$1 increases in rent fee

$R_{initial} = \$30 \cdot 500$

$R(n) = (30 + 1n)(500 - 10n)$

$= 15000 - 30n + 500n - 10n^2$

$R'(n) = -30 + 500 - 20n = 0$

$20n = 470$

$n = 23\frac{1}{2}$ increases of \$1 above \$30

$\$30 + \$1(23.5) = \boxed{\$53.50 \frac{\text{rental}}{\text{day}} \text{ max revenue}}$

8. $Y(n) = b(n) \cdot t(n)$

where Y is yield, b is $\frac{\text{bushels}}{\text{tree}}$, t is $\frac{\text{trees}}{\text{acre}}$

n is # of additional trees per acre

$Y_{initial} = 30 \cdot 20$, $Y(n) = (30 - 1n)(20 + 1n)$

$= 600 + 10n - n^2$

$Y'(n) = 10 - 2n = 0$, $\boxed{n = 5 \text{ more trees or } 25 \text{ trees/acre}}$

9a) $R(n) = q(n) \cdot p(n)$, $\quad$ hot dogs $\cdot \frac{\$}{\text{hot dog}} = \$$ revenue

$R_{\text{initial}} = 10{,}000 \cdot \$4$

Write $R(n)$ where $n = \#$ of 25¢ price increases from starting pt of 10,000 and \$4

Each \$.25 ↑ causes 500 hot dogs ↓

$R(n) = (10{,}000 - 500n)(4 + .25n)$

$= 40{,}000 + 500n - 125n^2$

$R'(n) = 500 - 250n = 0 \longrightarrow n = \frac{500}{250} = 2$

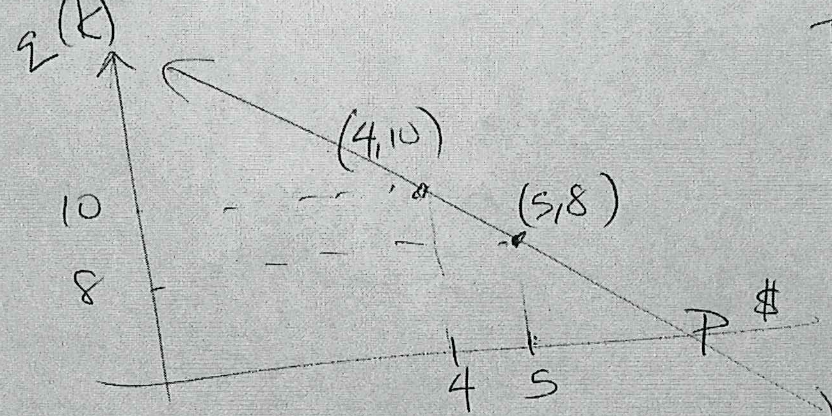
$n = 2$ price increases of 25¢ each

So $p(n) = 4 + .25(2) = 4 + .50 = \boxed{\$4.50}$

b) Using a linear model (we know the change in price gives a constant change in sales, i.e. slope is const), Choose two pts. not too close (to make it easier). The fcn will be

$p(q) = m \cdot q + b$ or $p - p_1 = m(q - q_1)$

I'll use a scale of 1000 hot dogs to also make things easier to read + graph



From \$4 to \$5 represents a loss of 4(500) hot dogs, or $10K - 2K = 8K$

$$m = \frac{8-10}{5-4} = \frac{-2K}{\$1}$$ which is correct!

Using pt-slope model:

$$q - 10 = -2(p-4)$$

$$q(p) = -2p + 18$$ quantity fcn.

$$R(p) = p \cdot q(p) = p(-2p+18)$$

$$R = -2p^2 + 18p$$

$$R' = -4p + 18 = 0 \longrightarrow p = \frac{18}{4} = \boxed{\$4.50}$$

As before!

Optimization – Focus from each category

1. $C(x) = 15{,}000 + 40x + 0.02x^2$

$R(x) = 100x - 0.01x^2$

$P(x) = R(x) - C(x) = -15{,}000 + 60x - .03x^2$

$P'(x) = 60 - .06x = 0$, $\boxed{x = \frac{60}{.06} = 1000 \text{ units}}$

2. $C(x) = 12x$, $x(p) = 50 - p$

(alternate notation: $C(q) = 12q$
$q(p) = 50 - p$)

q is a fcn of p as given, but
C is a fcn of q as given.

Composite fcn $C(q(p)) = 12(50-p)$

That is, $C(p) = 12 \cdot 50 - 12 \cdot p = 600 - 12p$

and $R(p) = q \cdot p = (50-p)(p)$
$= 50p - p^2$

$P(p) = (50p - p^2) - (600 - 12p) = -600 + 62p - p^2$

$P'(p) = 62 - 2p = 0$, $\boxed{p = \$31}$

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