Sec 23 Optimization

What does it mean to maximize revenue or profit? Recall that marginal functions tell us the cost or revenue for producing or selling the $(x+1)$th unit. And marginal fcns. are the derivative of the given fcn.

Given $R(x)$, to find marginal revenue at $x = 10$, for ex, calculate $R'(10)$. This gives the revenue obtained by selling the 11th item. Also, recall, this is an approximation: $R'(10) \approx R(11) - R(10)$.

When revenue starts dropping off, marginal revenue gets smaller. At a certain level of sales, the $(x+1)$th unit brings in no further revenue. That level will be where $R'(x) = 0$.

Thus, we maximize revenue where $R'(x) = 0$. Usually there is more to this problem than just an equation as given. (See video clip) For ex, book example 23.5 is more typical than 23.1.

Volume + surface area problems are also quite typical (book ex. 23.2–23.4)

Video

The video clip variable n = # of increases in price and so the increase term is $.25n$.
The loss in sales is 500, so the decrease term is $500n$.

$$R(n) = p(n)\,q(n)$$
$$= (4.00 + .25n)(10{,}000 - 500n)$$

Book Ex 23.5 $\quad x =$ number of \$1 rebates

$$R(x) = p(x)\,q(x)$$
$$= \underbrace{(\$450 - \$1x)}_{\text{price goes down \$1 for each \$1 rebate}}\underbrace{(1000 + 10x)}_{\text{sales go up by 10 for each \$1 rebate}}$$

Multiplying these:

$$R(x) = 450{,}000 + 4500x - 1000x - 10x^2$$
$$R(x) = 45{,}000 + 3500x - 10x^2$$

$$R'(x) = 3500 - 20x = 0 \quad \text{at} \quad x = 175$$

or \$175 rebate ensures maximum profit for this revenue fcn.

$$C(q) = 68{,}000 + 150q$$

Cost is given as a fcn. of q, but q is related to the rebate on a weekly basis by the relationship:

$$q = 1000 + 10x$$

So $C(q(x)) = 68{,}000 + 150(1000 + 10x)$

$= 68{,}000 + 150{,}000 + 1500x = 218{,}000 + 1500x$

and $P(x)$ as usual is $R(x) - C(x)$:

$$P(x) = R(x) - C(x)$$

$$= 450{,}000 + 3500x - 10x^2 - (218{,}000 + 1500x)$$

$$= 232{,}000 + 2000x - 10x^2$$

$P'(x) = 2000 - 20x = 0$ at $x = 100$ one dollar rebates ($100)

What is the profit / week at this value?

$P(x) = P(100) = R(100) - C(100)$

$= 232{,}000 + 2000(100) - 10(100)^2$

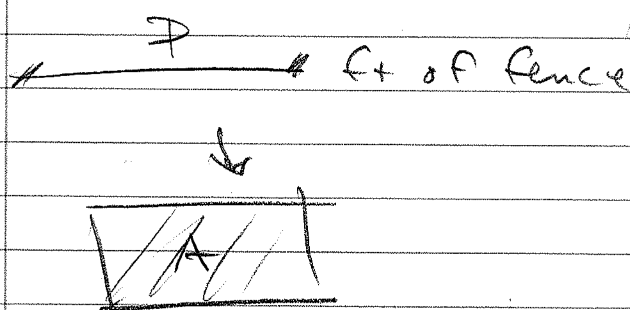
100,000

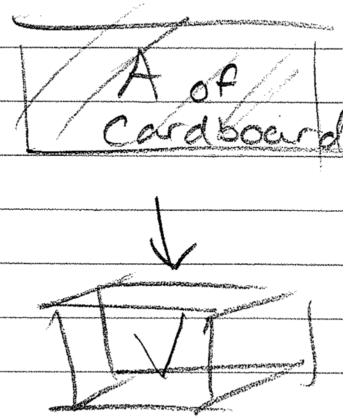
$= \$332{,}000$

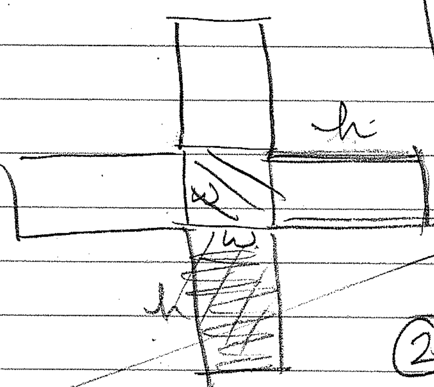
Geometric problems in optimization

can box 3D entail vol formula & surface area formula OR

garden plot 2D entail area & perimeter formulas







① Vol = lwh

$\boxed{32{,}000 = w^2 h}$

② Surface area of open box

$\boxed{SA = 4\underline{w}\underline{h} + w^2}$

① Let $\boxed{h = \frac{32{,}000}{w^2}}$

$SA = 4\cancel{w}\left(\frac{32{,}000}{w^{\cancel{2}}}\right) + w^2$

$SA(w) = \frac{128{,}000}{w} + w^2$

$SA'(w) = -128{,}000w^{-2} + 2w = 0$

$= -\frac{128{,}000}{w^2} + \frac{2w \cdot w^2}{w^2} = 0$

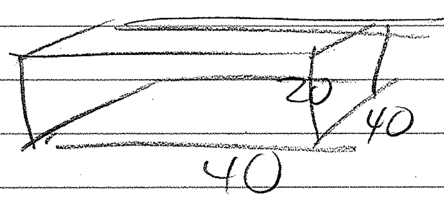
$$\frac{-128{,}000 + 2w^3}{w^2} = 0$$

$$w^3 = \frac{128{,}000}{2} = 64{,}000$$

$$\boxed{w^3 = 64{,}000} \longrightarrow w = 40$$

$$h = \frac{32{,}000}{w^2} = \frac{320\cancel{0}\cancel{0}}{16\cancel{0}\cancel{0}} = 20\text{cm}$$

Box $20 \times 40 \times 40$



Sec 23 - Optimization HW

1. $x + y = 150$, $x^2 y = x^2(150 - x) = 150x^2 - x^3$

$y = 150 - x$ $\qquad f(x) = 150x^2 - x^3$

$x = 100$, $y = 50$ $\qquad f'(x) = 300x - 3x^2 = 3x(100 - x) = 0$

$\cancel{x=0}$, $x = 100$

What is the maximum product $x^2 y$?

$100^2 \cdot 50 = 50,000$

2. $lw = A$ w l Area $lw = 3600 \rightarrow l = \frac{3600}{w}$

Perimeter $2(l+w) = 2\left(\frac{3600}{w} + w\right) = \frac{7200}{w} + 2w$

$P(w) = \frac{7200}{w} + 2w$, $P'(w) = -7200w^{-2} + 2 = 0$

$-\frac{7200}{w^2} = -2 \rightarrow w^2 = 3600$

$w = 60\,m$

$A = lw = l(60) = 3600\,m^2$

$l = \frac{360\cancel{0}}{6\cancel{0}} = 60\,m$

4. $p(x) = 4 - \frac{x}{12}$, x thousand items

$R(x) = x \cdot p(x) = x\left(4 - \frac{x}{12}\right) = 4x - \frac{x^2}{12}$

$R'(x) = 4 - \frac{2x}{12} = 4 - \frac{x}{6} = 0 \rightarrow 24 - x = 0$, $x = 24$

max rev. $R(24) = 24\left(4 - \frac{24}{12}\right) = 24(4-2) = 48$ $\boxed{\$48,000}$

5. $P'(x) = R'(x) - C'(x)$, since $P(x) = R(x) - C(x)$

Also, $R'(x) - C'(x) = 0$, when $R'(x) = C'(x)$

$P'(x) = 70 - x - .1x^2 - 4x - 10 = 60 - 5x - .1x^2 = 0$

that is, $600 - 50x - x^2 = (60 + x)(10 - x) = 0$

$\cancel{x = -60}$, $x = 10$

6. Given $p(x)$, $C(x)$, and $R(x)$ here: $p(x) = 10 - \frac{x}{400}$

$C(x) = 400 + 2x + .05x^2$, $R(x) = x \cdot p(x) = x\left(10 - \frac{x}{400}\right)$

Find prod. level x to maximize profit.

$R(x) = 10x - \frac{x^2}{400}$, $\boxed{R'(x)} = 10 - \frac{2x}{400} = 10 - \frac{x}{200}$

$\boxed{C'(x)} = 2 + .1x$, $P'(x) = R'(x) - C'(x) = 0$

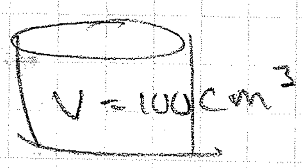
$P'(x) = 10 - \frac{x}{200} - 2 - .1x =$

$= 8 - \frac{x}{200} - \frac{x}{10} = 8 - \frac{x - 20x}{200}$

$= \frac{1600 - 21x}{200} = 0 \rightarrow 1600 - 21x = 0$

Prod level: $x = \frac{1600}{21}$ units ≈ 76 units

7.



$h = ?$
$r = ?$
$V = 100$

Know formula $V = \pi r^2 h$ and that $V = 100$

Substit: $100 = \pi r^2 h$

12. $p = \$30$ $q = 120/\text{mo}$ $p(x) = 30 - 2x$ $q(x) = 120 + 10x$
where x = number of price drops

$R(x) = p(x)\,q(x) = (30 - 2x)(120 + 10x)$

$R(x) = 3600 + 60x - 20x^2$

$R'(x) = 60 - 40x = 0 \rightarrow x = \frac{3}{2}$ \$2 price drops

$\frac{3}{2} \cdot 2 = \$3$ reduction maximizes revenue

Sell lamp for $\$30 - 3 = \$27$