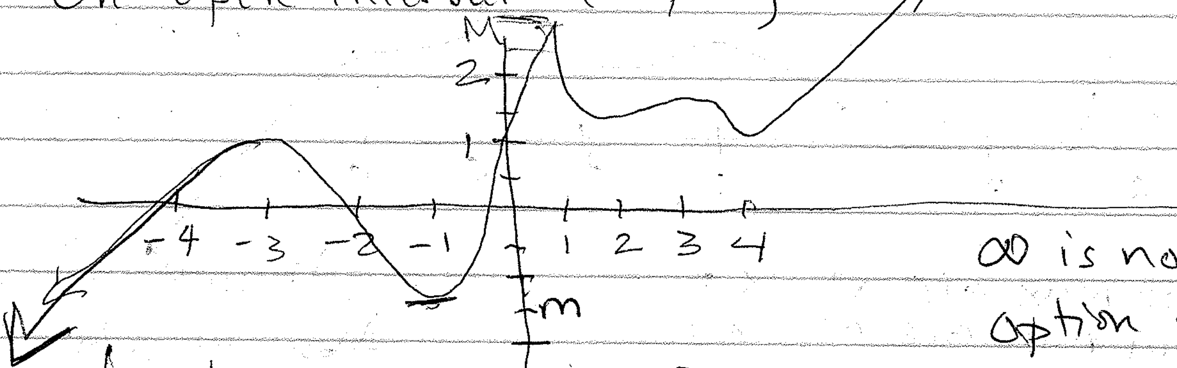
Sec. 21 Absolute Extrema (Maximum + Minimum)

On open interval $(-\infty, \infty)$



∞ is not an option for an abs max M or min m

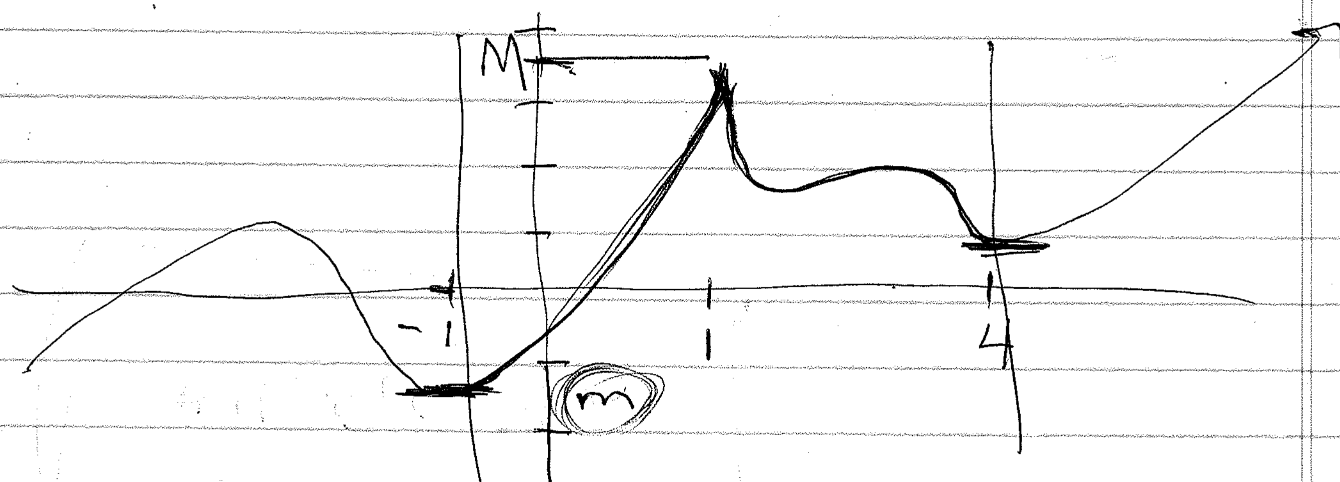
Local max at $x = -3, 1, 3$

Local min at $x = -1, 2, 4$

Absolute max ~~none~~ none — no number M such that $f(x) > M$ on $\mathbb{R}$

Absolute min

On closed interval $[a, b] = [-1, 4]$

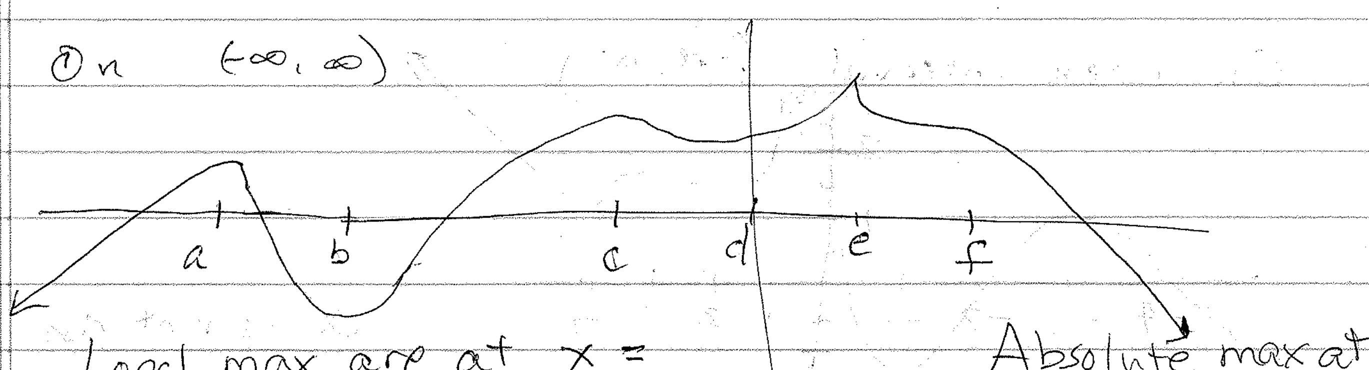


Remember, the extreme value theorem says that a cts. fcn. f on a closed interval has both a greatest value M and a least value m. $M \equiv \max$

$m \equiv \min$

Section 22 Absolute Maxima & Minima

① On $(-\infty, \infty)$



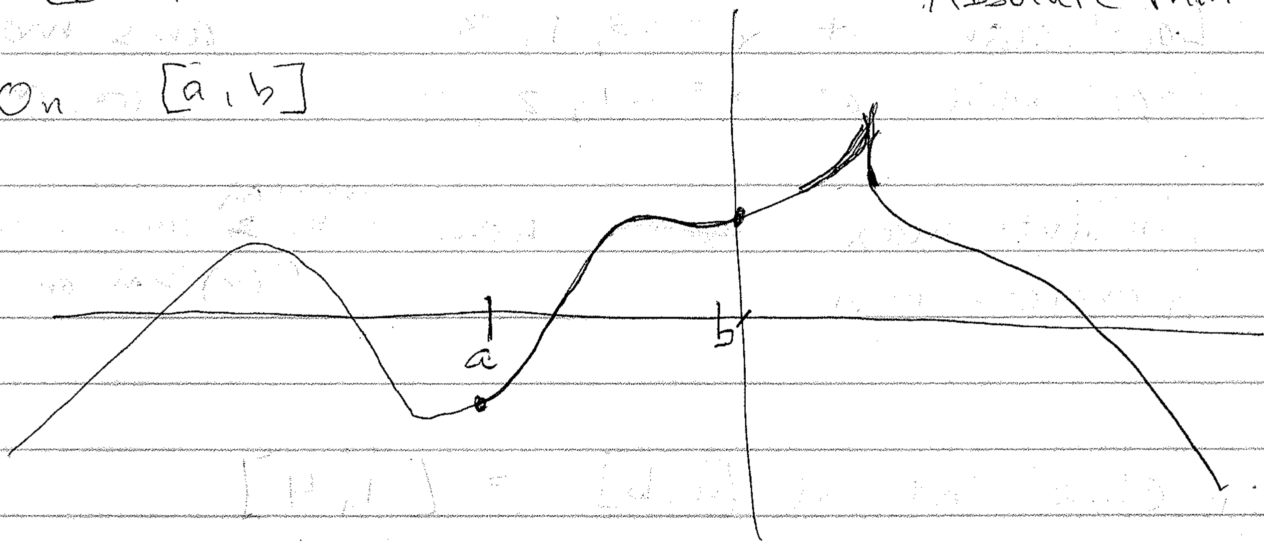
Local max are at $x =$

Local min are at $x =$

Absolute max at

Absolute min

On $[a, b]$



#3 a, c, d

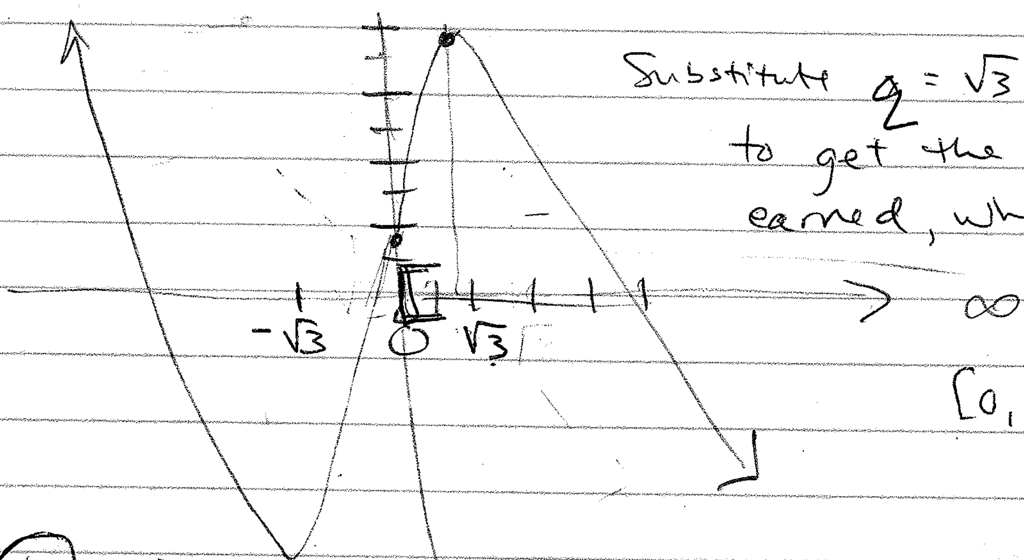
#4, 5, 6, 7

Ex #22.6 in book

$P(q) = -q^3 + 9q + 3$ Dom: $[0, \infty)$

$P'(q) = -3q^2 + 9 = 0$ $q = \pm\sqrt{3}$

$P''(q) = -6q$, $P''(\sqrt{3}) = -6\sqrt{3} < 0$, c. down



Substitute $q = \sqrt{3}$ into $P(q)$ to get the max profit earned, where q = quant

$P(q)$ = profit

$[0, \infty)$

Compute: $P(\sqrt{3}) = -3\sqrt{3} + 9\sqrt{3} + 3 = 6\sqrt{3} + 3 \approx 15$ est

$P(-\sqrt{3}) = 3\sqrt{3} - 9\sqrt{3} + 3 = -6\sqrt{3} + 3 \approx -10$ est

$P(q) = -q^3 + 9q + 3$

$P(\sqrt{3})$ is local & abs max

P(root) abs max

To find the absolute maximum and minimum of a continuous fcn. f on closed interval $[a,b]$:

1. Find all values of f at critical numbers of f in (a,b)
2. Find the values of f at the endpoints of the interval
3. Of all the values in 1 & 2, the largest is the absolute maximum, the smallest is is the absolute minimum.

Ex #22.5 in book $f(x) = \frac{1-x^2}{x^3}$ on $[1,\infty)$

Find all critical numbers of f and determine the local and absolute extremes.

Natural Domain: $(-\infty,0)\cup(0,\infty)$ But given $[1,\infty)$

~~VA: $x=0$~~

HA? deg of numerator < deg of denominator so there's an HA at $y=0$

$$f'(x) = \frac{(-2x)(x^3)-(1-x^2)(3x^2)}{x^6} = \frac{x^4-3x^2}{x^6}$$

$$f'(x) = \frac{\cancel{x^2}(x^2-3)}{x^{\cancel{6}\,4}}$$

$x=0, +\sqrt{3}, -\sqrt{3}$ are crit #s in $[1,\infty)$

$$f''(x) = \frac{(2x)(x^4)-(x^2-3)(4x^3)}{x^8} = \frac{-2x^5+12x^3}{x^8}$$

$$f''(x) = \frac{12-2x^2}{x^5}$$

Evaluate f'' at $\pm\sqrt{3}$ →