1

Sec 17 - #3 Cost fcn $C(x) = 2x^3 + 3x^2 + 6x + 24$

x = quantity in hundreds of liters
Find intervals of increase + decrease of $C(x)$

By theorem 1, if I is an open interval where $C'(x) > 0$, then $C(x)$ is increasing on I. Likewise for $C'(x) < 0$, then $C(x)$ is decreasing on I.

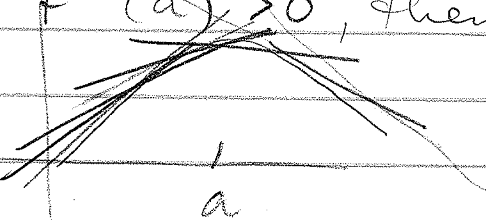
$\boxed{C'(x) = 6x^2 + 6x + 6}$ Since domain of $C(x)$ is $x \geq 0$ (for a polynomial, dom is $\mathbb{R}$, but only nonnegative x makes sense in production), and since all terms are positive in $C'(x)$, $C(x)$ increases everywhere.

Another way to view this is to consider that the derivative is a parabola with no real roots ($C'(x) = 6x^2 + 6x + 6$; the discriminant $b^2 - 4ac$ is less than zero, so the parabola is above the x-axis) You get complex roots, and there's no way to plot these on the x-axis and look at intervals on "either side". Complex numbers do not lie on the real number line.

Anticipating curve sketching $C(x) = 2x^3 + 3x^2 + 6x + 24$

- Knowing C increases everywhere, we can only rely on the x- + y-intercepts and concavity + points of inflection.

2nd derivative test This says that if $f'(a) = 0$ and $f''(a) < 0$, then f has a local maximum at a. Likewise, if $f''(a) > 0$, then f has a local minimum at a.



But our fcn $C(x)$ has no value a where $f'(a) = 0$.

Even though it has no critical point, it could still have an inflection point. By definition, a function has a point of inflection where $f''(a) = 0$.

Ex $f(x) = \frac{1}{3}x^3 + \frac{1}{2}x^2 + x$ Dom $\mathbb{R}$

Find x + y intercepts first to get an idea of this graph. Also, consider end behavior. Odd degree + positive lead coeff

$f(0) = 0$ and there is no other x where $f(x) = 0$.

$$f(x) = x\underbrace{\left(\tfrac{1}{3}x^2 + \tfrac{1}{2}x + 1\right)}_{\text{This portion is a parabola with no roots}}$$

$x = 0$

Find $f'(x)$:

$f'(x) = x^2 + x + 1 \neq 0$ Also, no roots.

from $f(x) = \frac{1}{3}x^3 + \frac{1}{2}x^2 + x$

(3)

What do we do when a fcn. has complex roots? Conclude right away that it is a parabola that doesn't intersect x-axis

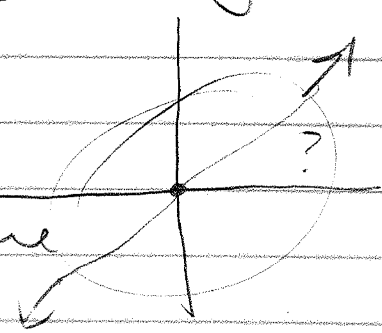
But what about critical points?
By <u>definition</u>, if $f'(c) = 0$ or DNE, c is a critical number.

$f'(x) = x^2 + x + 1 = 0$ when $x = \frac{-1 \pm \sqrt{-3}}{2}$
That is, no roots. $f'(x) > 0$ for all x
so f is <u>increasing everywhere</u>.

However, $f(x)$ could have an inflection point.

<u>Def</u> If $f''(c) = 0$ then f has an <u>inflection pt</u> at c (concavity changes)

Our example still $\boxed{f(x) = \frac{1}{3}x^3 + \frac{1}{2}x^2 + x}$



It has one root and it's increasing everywhere but it isn't a line.

So something is happening with concavity.

$f'(x) = x^2 + x + 1$
$f''(x) = 2x + 1 = 0$ when $x = -1/2$

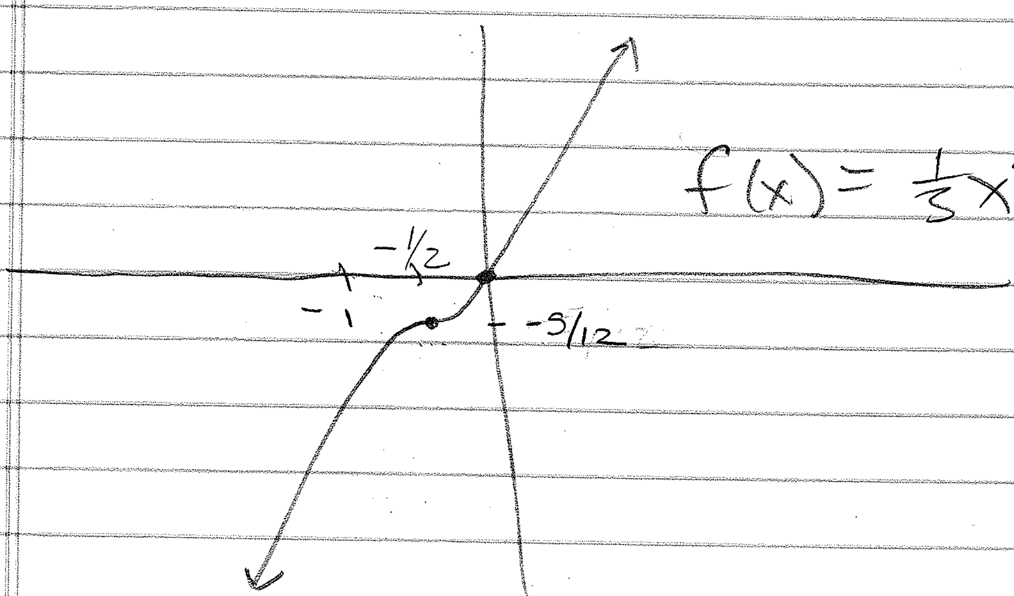
Point of inflection $(-1/2, f(-1/2))$

4

$$f(-1/2) = \frac{1}{3}\left(-\frac{1}{2}\right)^3 + \frac{1}{2}\left(-\frac{1}{2}\right)^2 + \frac{-1}{2}$$

$$= -\frac{1}{24} + \frac{1}{8} - \frac{1}{2} = -\frac{5}{12}$$

$$f(x) = \frac{1}{3}x^3 + \frac{1}{2}x^2 + x$$



Now look at concavity. Since $x = -1/2$ is site of inflection, we ought to see a change in sign of $f''(x)$ at $x = -1/2$, signalling a change in concavity.

Test a value left and right of $x = -1/2$ into the second derivative:

$$f''(-1) = 2(-1) + 1 = -1 < 0$$

Conclude f is concave down to left of $-1/2$ since negative second derivative indicates the slope of tangent is decreasing.

$$f''(0) = 2(0) + 1 = 1 > 0$$ Conclude f is concave up to right