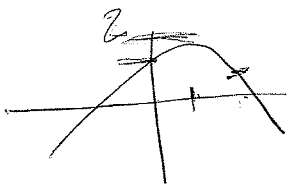
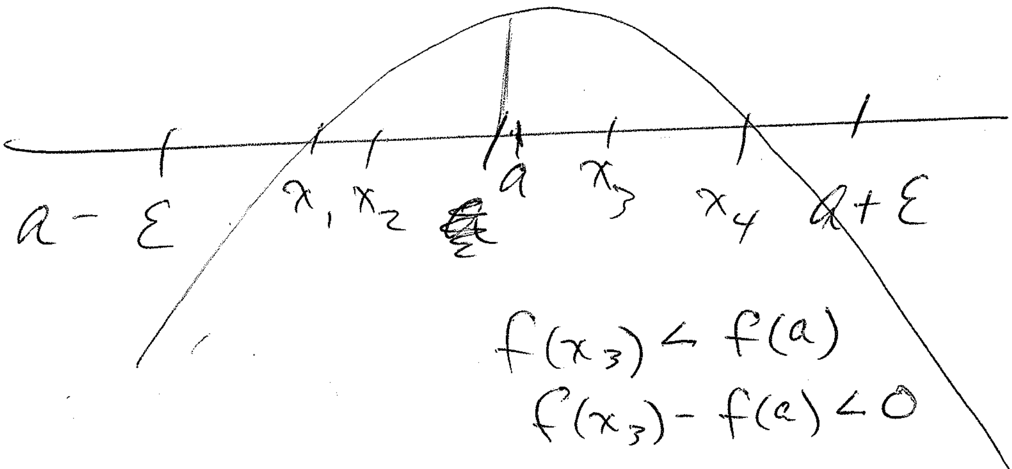
Def of local extema

Def of local max at $x=a$

$f(a)$ is a local maximum of the fcn $f(x)$ if in an ε n'hood of "a"



$f(x) - f(a) < 0$

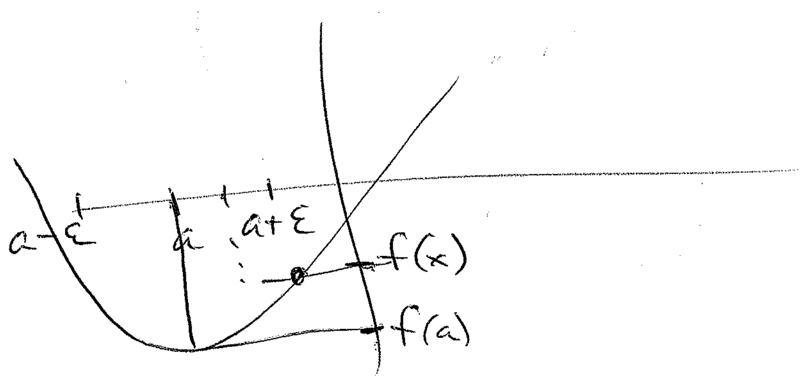


$f(x_3) < f(a)$

$f(x_3) - f(a) < 0$

Def of local min at $x=a$

$f(a)$ is a local minimum of $f(x)$ if in an ε- n'hood of "a", $f(x) - f(a) > 0$

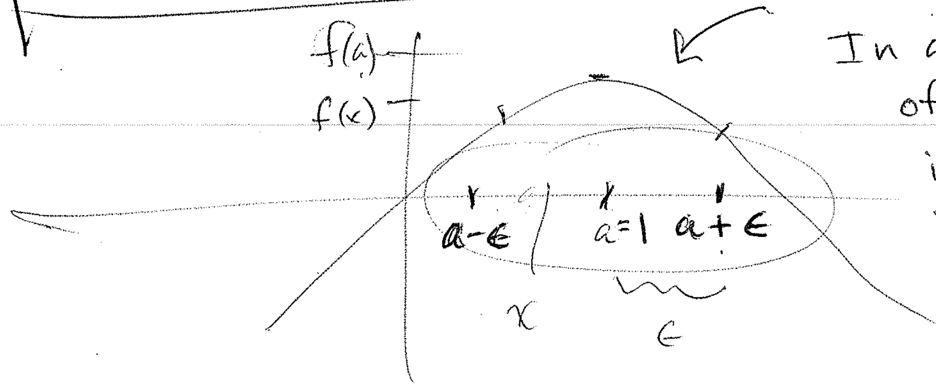


Sec 15 Local Max + Min

Extrema

Local Maximum

$$\boxed{f(x) - f(a) < 0 \quad \text{whenever} \quad |x-a| < \epsilon}$$

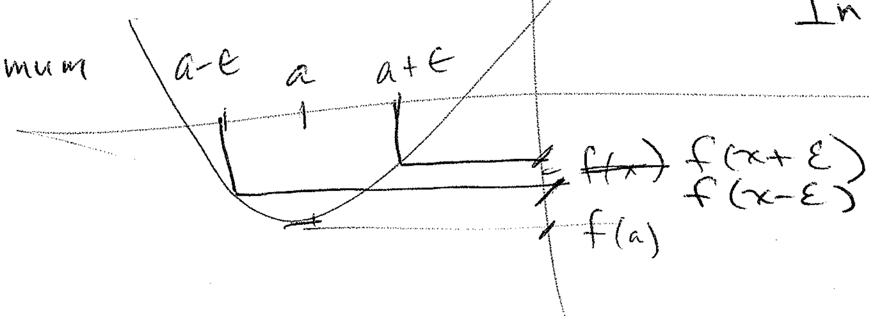


In an epsilon neighborhood of "a", the function is never larger than it is at $x=a$ itself.

$\boxed{f(1) \text{ is a local max of the fcn}}$

$f(x) - f(a) > 0$ whenever $|x-a| < \epsilon$

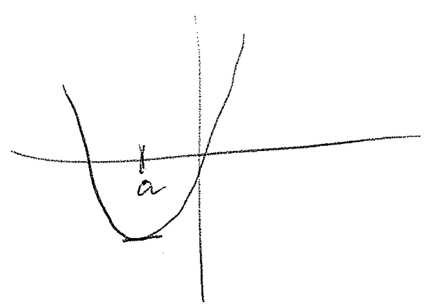
Local Minimum



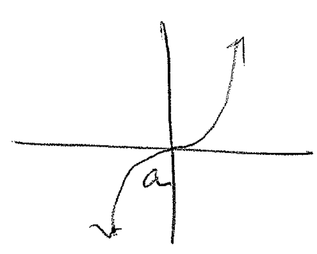
In an epsilon neighborhood of "a", the fcn. is never smaller than it is at $x=a$ itself.

If $f(x)$ has a loc max or min at $x=a$, ~~then~~
and if it's diff'ble at $x=a$
then $f'(a)=0$

Extremum at $x=a$ $\rightarrow$ $f'(a)=0$



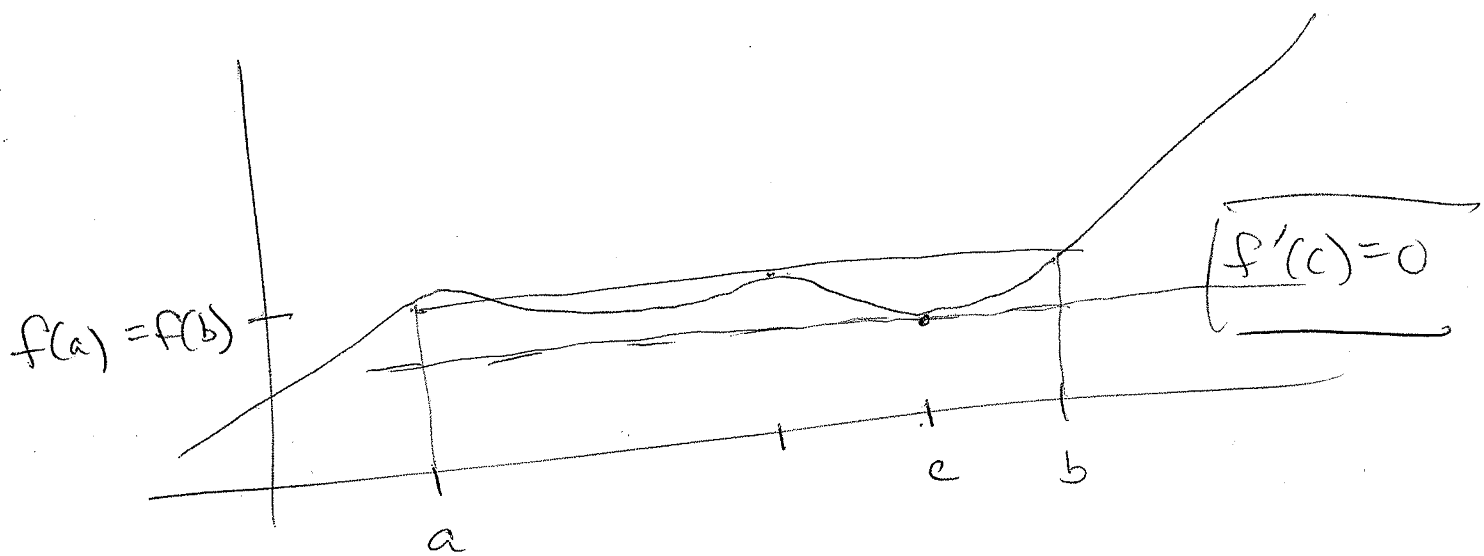
$f'(a)=0 \not\rightarrow$ Extremum at $x=a$



Rolle's Theorem (special case of MVT)

Suppose f is cts on $[a,b]$ and differentiable on (a,b) and $f(a) = f(b)$.

Then there is at least one c in (a,b) at which $f'(c) = 0$



Special case of MVT because if $f(b) = f(a)$, then $f(b) - f(a) = 0$, so the mean value of f on $[a,b]$ is $\frac{0}{b-a} = 0$.

~~Extra~~

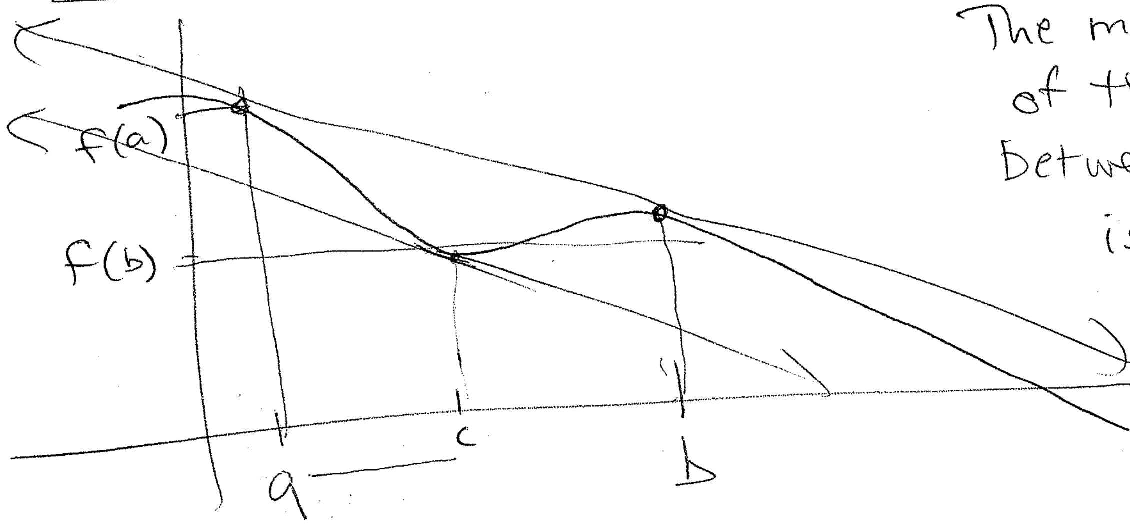
Mean Value Thm (MVT)

Suppose f is cts. on $[a,b]$, and suppose f is differentiable on (a,b), and (so the endpts won't mess us up)

$f(a) \neq f(b)$.

Then there is at least one ~~point~~ value c in (a,b) at which $\boxed{f'(c) = \dfrac{f(b)-f(a)}{b-a}}$

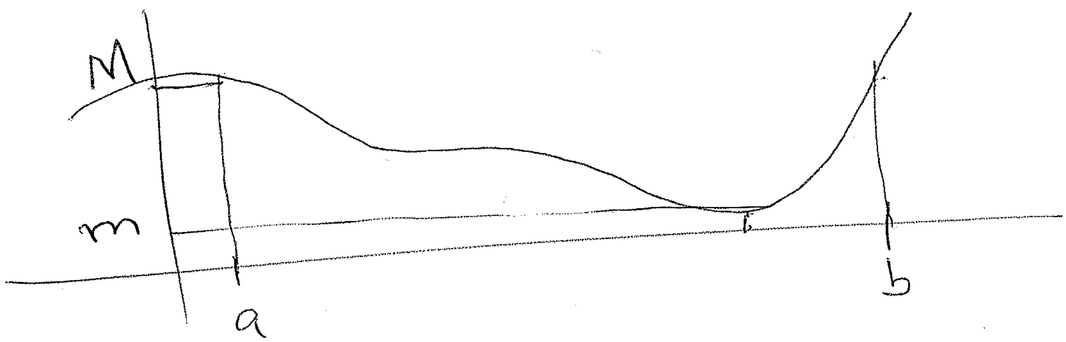
or $\boxed{f(b)-f(a) = f'(c)(b-a)}$



The mean value of the fcn between a & b is $\dfrac{f(b)-f(a)}{b-a}$

Extreme Value Thm (EVT)

Suppose f is cts on $[a,b]$.
Then f has both a maximum M and a minimum m on $[a,b]$.

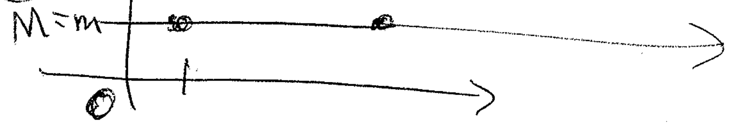


That is, there is a number c on $[a,b]$ where $f(c)$ is an absolute maximum and a number d on $[a,b]$ where $f(d)$ is an absolute minimum.

These c, d could be a or b themselves.

Def Absolute (global) min or max
(An absolute min (m) or max (M) is the value of the fcn on $[a,b]$ ~~where that is less than~~

where $f(x) \leq M$ for all x in $[a,b]$
and $f(x) \geq m$ for all x in $[a,b]$)



Sec 16 IVT, EVT, MVT (Rolle's Thm)

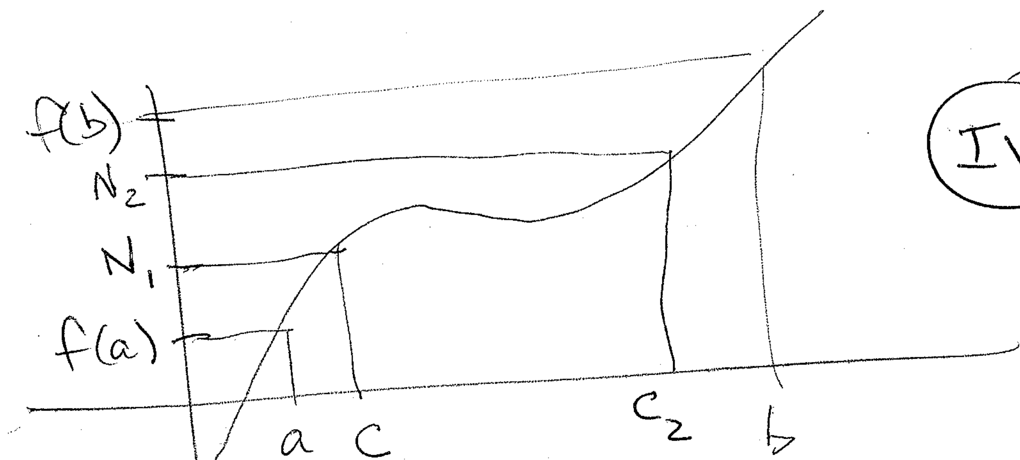
Intermediate Value Thm (IVT)

Suppose f is cts on $[a,b]$ and $f(a) \neq f(b)$

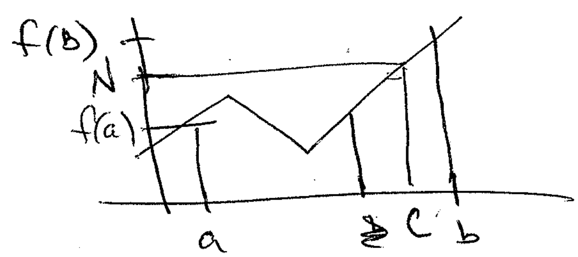
Then, if N is some number between $f(a)$ + $f(b)$

there must be at least one number c

between a + b for which $f(c) = N$



IVT



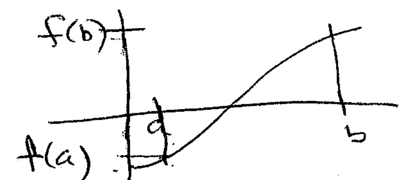
Corollary to IVT

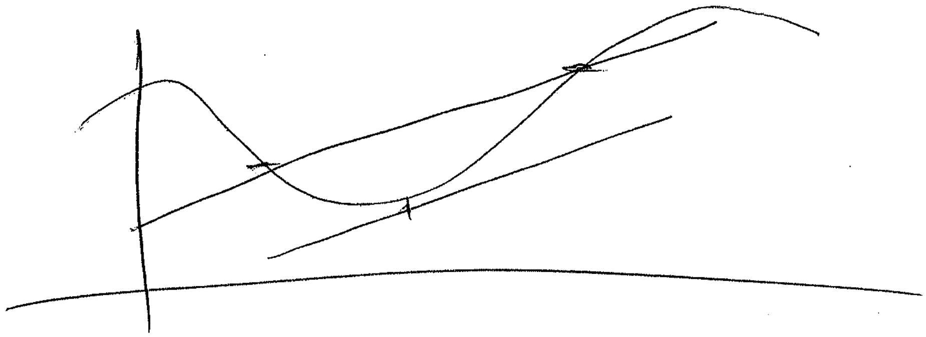
Suppose f is cts on $[a,b]$ and $f(a) < 0 < f(b)$

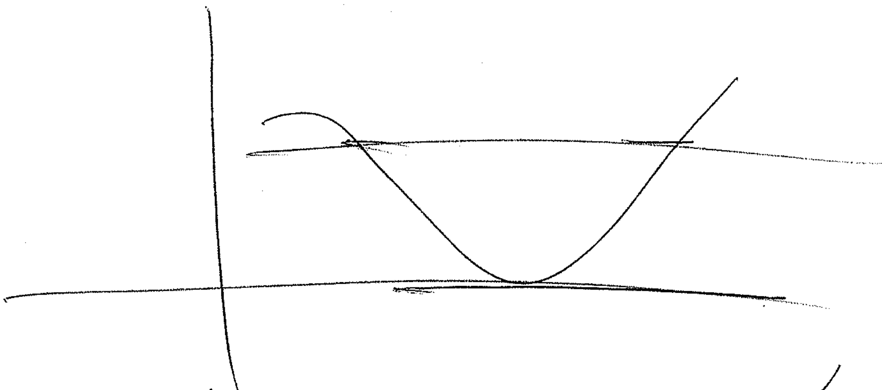
(i.e. 0 lies between $f(a)$ + $f(b)$)

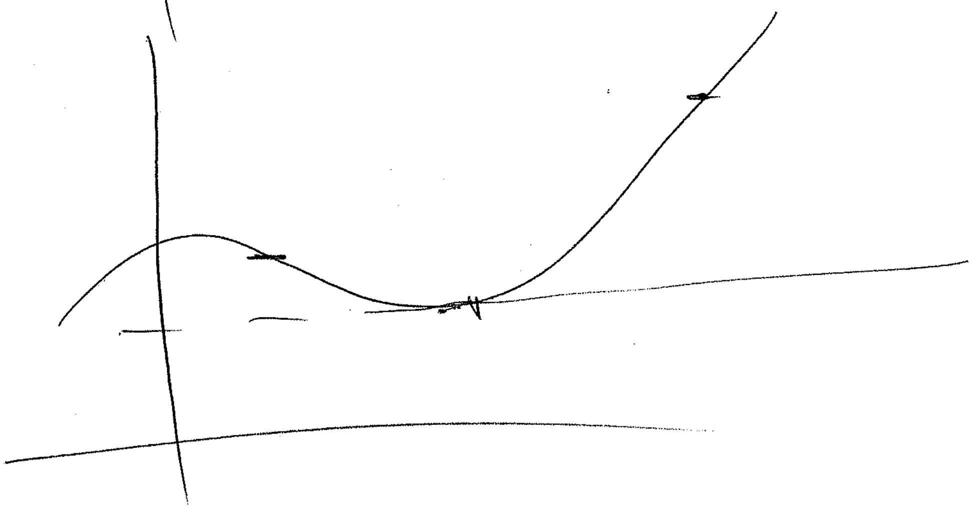
Then there must be at least one number c

between a + b for which $f(c) = 0$

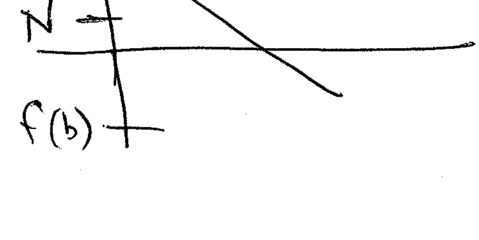








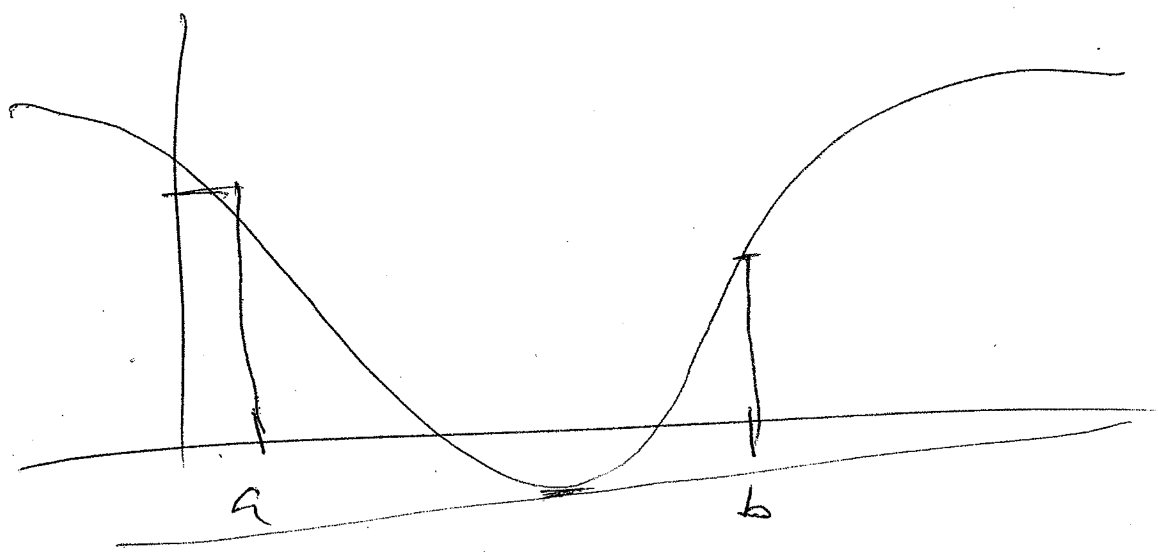
IVT + EVT are not concerned with derivative



Rolle's Thm is the special case of the MVT

general $f(a) \neq f(b)$ MVT

special $f(a) = f(b)$ Rolle's



It could happen that if $f(a) \neq f(b)$, we still get a c where $f'(c) = 0$

But ~~it does~~ it is not sufficient

$f(a)=f(b)$

Does the const fcn adhere to the const conditions of Rolle's Thm?

(1) f cts (2) f is diff (3) $f'(c) = 0$ all x