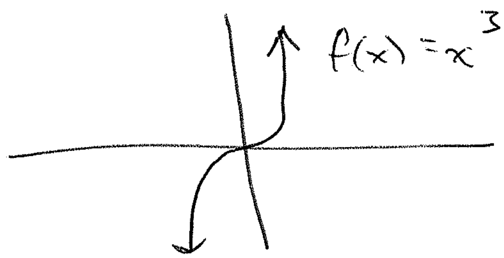
Thm If $f(x)$ is differentiable on its domain and if it has a local max (min) at a, then $f'(a) = 0$

The converse is not true. If $f'(a) = 0$ then $f(x)$ does not necessarily have a local max (min) at a.

Counterexample $f(x) = x^3$ differentiable on $\mathbb{R}$ (its domain)

$f'(x) = 3x^2 = 0$ at $x = 0$.

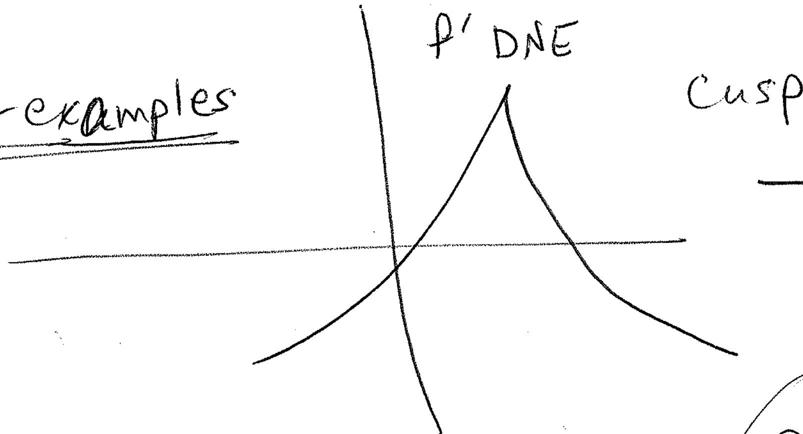
But clearly $f(0)$ is not a local extreme.

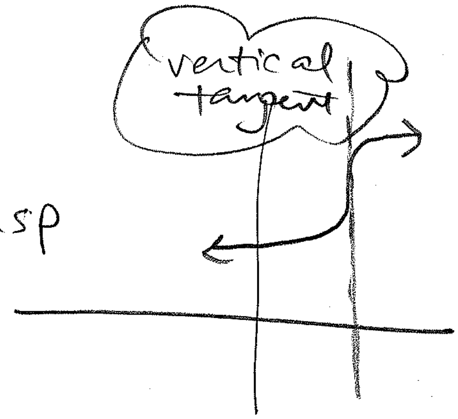


Why not? Because, by def of local max (min) "a is a local max (min) if for a neighborhood $\varepsilon > 0$ of a, $f(x) \leq f(a)$ when $x \in (a-\varepsilon, a+\varepsilon)$ for the local max and $f(x) \geq f(a)$ for the local min situation when $x \in (a-\varepsilon, a+\varepsilon)$

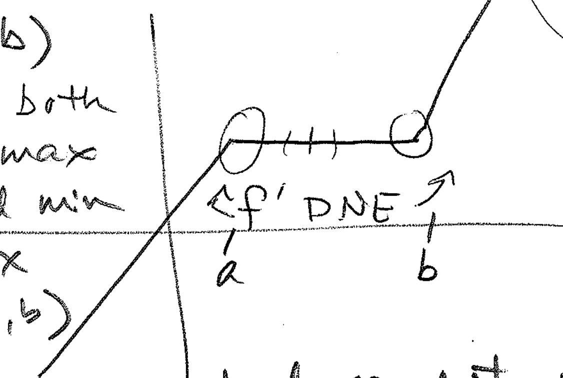
So, to complete the understanding of the theorem, whenever f has a local extremum at $x=a$ it is not necessarily true that f is differentiable at a.

Counterexamples





On (a,b) $f(x)$ is both a local max & a local min for all x in (a,b)



$f(x) \le f(a)$ max
$f(x) \ge f(b)$ min

corner

Rarefied

Key - The interval does not include $x=a$ or b

So, if f has a local extreme at a, then either $f'(a)=0$ or DNE.

Wrap for $f(x) = \text{constant}$

$f'(x) = 0$ for all x

and $f(x)$ is both local max + min for all x

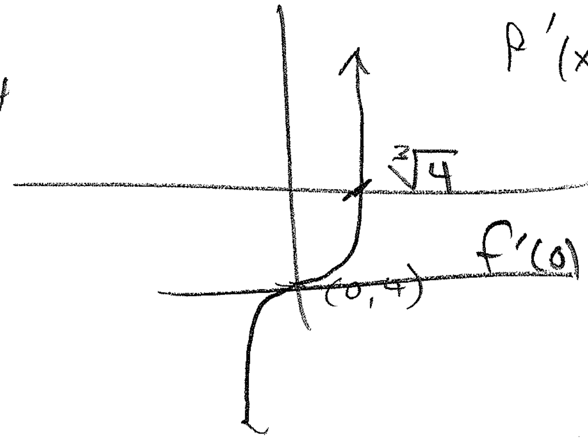
$f(x) = x^3 - 4$

$f'(x) = 3x^2$

$x^3 - 4 = 0$

$x^3 = 4$

$x = \sqrt[3]{4}$



$f'(0) = 0$

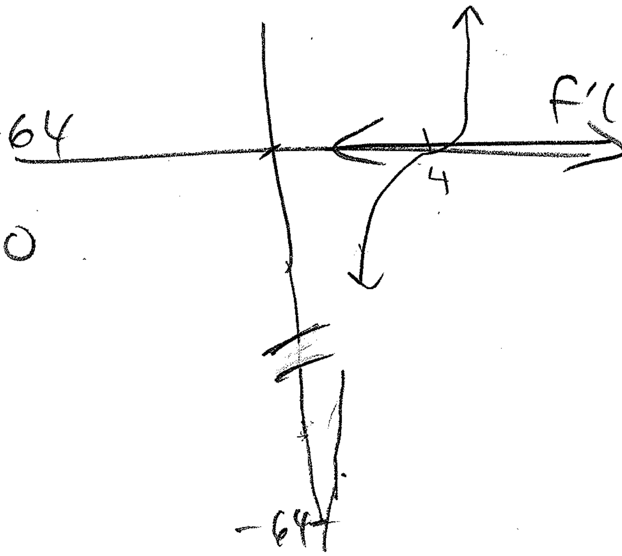
$\boxed{f'(x) = 0 = 3x^2 \quad x = 0}$

$g(x) = (x-4)^2$

$g(0) = (0-4)^3 = -64$

$g'(x) = 3(x-4)^2 = 0$

$x =$

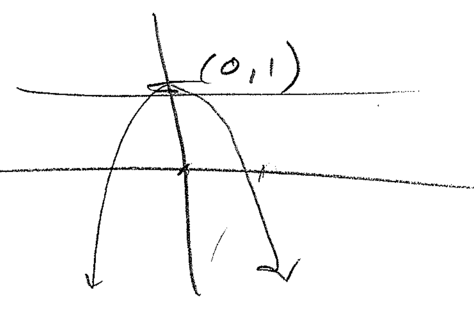


$f'(4) = 0$

$h(x) = -x^2 + 1$

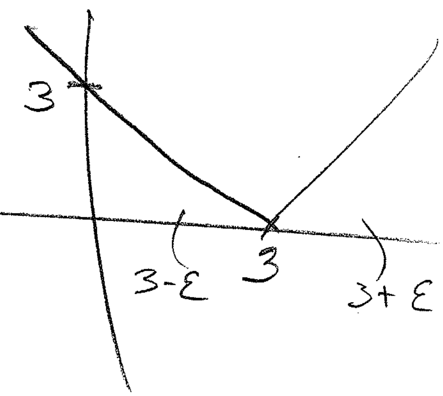
$h'(x) = -2x = 0$

at $x = 0$



Local max at $x = 0$

$\rightarrow f(x) = |x - 3|$



Local min at $x = 3$

But $\underline{f'(3) \text{ DNE}}$